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Review

Optimum stacking sequence design of composite materials Part II: Variable stiffness design

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ABSTRACT

A composite laminate may be designed as a permutation of several straight-fiber layers or as a matrix embracing fibers positioned in curvilinear paths. The former called a constant stiffness design and the latter known as variable stiffness design. The optimization algorithms used in constant stiffness design were studied in Part I of this review article. This paper completes the previous article by focusing on variable stiffness design of composite laminates. Different parameterization and optimization algorithms are briefly explained and compared and the advantages and shortcomings of each algorithm are discussed.

Contents

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1. Introduction

Designing a laminated composite material consists in selecting the best arrangement of the constituent materials within the laminate. Traditionally, this task is performed by finding a combination of several straight-fiber layers with constant thicknesses such that the combination provides the best mechanical properties for a given application. However, allowing the fibres to follow curvilinear paths within the plane of laminate constitutes an advanced tailoring option that can lead to modification of load paths within the laminate and result in a more favourable stress distribution and an improved structural performance. We refer to the former approach as *constant stiffness design* while the latter is called *variable stiffness design*. Superior structural performance of variable stiffness design vs. constant stiffness design have been demonstrated for different properties such as buckling capacity [38,91,1], elastic behaviour [28], stiffness [89], compressive buckling and first-ply-failure [63], maximum fundamental frequency [12] and postbuckling progressive damage [62]. Variable stiffness design also provides flexibility for trade-off between different structural properties [29].

Variable stiffness design of composite structures has attracted far fewer researchers than constant stiffness design due to higher design and manufacturing costs involved. The higher design cost is due to the inordinately large number of design variables required to define variable orientations and thicknesses and additional constraints required for maintaining the continuity in the structure, which implies a need for higher computational resources compared to the constant stiffness design.

In the first part of this review, published in *Composite Structures* [25], we examined the constant stiffness design optimization of laminated composite materials. This paper, which completes the previous review article, focuses on the variable stiffness design of composite materials. First, the formulation of a variable stiffness design problem is studied and then related optimization methods are reviewed following this classification: (1) gradient-based methods, (2) optimality criterion, (3) topology optimization, (4) direct search methods, (5) multi-level optimization, and (6) hybrid methods. In each section, the essence of the main optimization algorithms is explained, followed by a discussion and comparison of their advantages and shortcomings.

2. Problem formulation and structural continuity

In contrast to the constant stiffness design, the design of variable stiffness composite structures requires a particular formulation that spatially defines the arrangement of the constituent materials (i.e. fibers and matrix) at each point of the structure. Regardless of the optimization technique, efficiency (i.e. convergence rate and accuracy) of the optimization algorithm is generally reduced as the number of design variables increases. On the other hand, independent design of these arrangements may result in an impractical structure with structural discontinuities. Hence, part of the effort in formulating a variable stiffness design problem is dedicated to minimizing the number of design variables in the problem formulation and to maintaining the continuity in the structure. For this purpose, three approaches have been introduced based on the use of (1) a patch design, (2) a set of blending rules, and (3) a curvilinear definition of the variable properties.

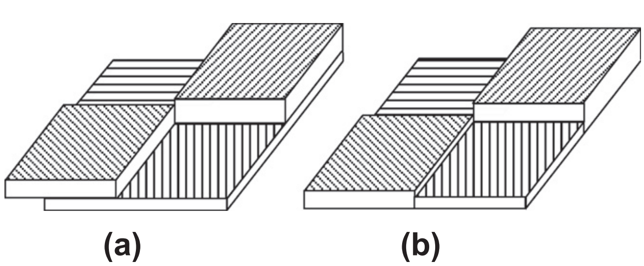

Fig. 1. A composite laminate divided into several overlapping (a) and non-overlapping (b) patches.

2.1. Patch design

A ‘‘patch’’ defines a region within the structure where the lamination sequence is uniform (see Fig. 1). The use of a patch allows reduction of the number of design variables and the spatial variation of the lamination sequences; consequently, it eases consideration of manufacturing limitations and maintains the compatibility of the lamination sequences in adjacent regions. Both overlapping patches [114,115] and non-overlapping patches [46] have been used to formulate a variable stiffness design problem. Usually in this formulation the patch geometry, which can strongly affect the efficiency of the final design, is defined *a priori* by the user.

The sub-laminate concept introduced by Soremekun et al. [97] can also be classified as a design with overlapping patches. A sub-laminate refers to the common thickness zones across multiple panels, and thus the identification of the sub-laminates is performed at the end of the optimization process, as oppose to the patch design where the patches are defined *a priori*. The similarity between the two methods is that in both methods the patch identification is performed by the user and it can be trained to reach either a high-performance design or a design with good blending properties.

2.2. Blending rules

Enforcing continuity constraints that limit the variation of the lamination sequences in adjacent elements is another method to achieve a blended structure. These constraints are generally known as blending rules. Examples of the blending rules are the ones used by Zabinsky et al. [113]. To design a tapered structure, they proposed blending rules stated as follows: ‘‘starting from the key region, the number of plies in adjacent regions may be dropped if the required stiffness and strengths of these regions are satisfied. Once a ply is dropped, it cannot be added back to the stacking sequence in later regions.’’ Kristinsdottir et al. [49] used the same blending constraints in design of sandwich and hat-stiffened panels. Liu and Haftka [58] developed a more sophisticated form of these rules by using continuity measures that distinguished between composition continuity and stacking sequence continuity. Although able to provide a required level of continuity in the structure, this method increases the number of constraints and makes the optimization problem more complex. To reduce the complexity of the design problem, it is possible to combine the blending rules with the patch design approach which can significantly reduce the number of design variables.

2.3. Curvilinear parameterization

The use of a curvilinear function to describe the fiber path and the variation of the laminate thickness can also reduce the number of design variables, without compromising structural continuity. The curvilinear function is identified by a set a parameters in a pre-defined mathematical expression [103] or by interpolating a pre-defined function to prescribed key points [36]. For instance, Parnas et al. [72] used Bezier splines to represent layer thicknesses and fiber angles. Blom et al. [12] used trigonometric functions to represent the fiber angles on a conical structure made by an advanced tow-placement machine. Nagendra et al. [70] expressed the fiber path as a linear combination of certain pre-defined base fiber paths. The design variables were the scalar multipliers of the different base fiber paths. Each base fiber path was a non-uniform rational B-spline curve passing through a fixed number of control points. Alhajahmad et al. [6] compared two cases where linear and non-linear functions were used to describe the variation of the fiber-orientation angles in a two-dimensional plate, and showed that the non-linear variations offered a better performance than linear variation of fiber orientations. Using a curvilinear function to describe the fiber path significantly reduces the number of design variables and guarantees the structural continuity; however, the quality of the final solution strongly depends on the parameterization. Finding a parametric function that can accurately model a complex structural geometry is difficult.

2.4. Other parameterizations

Besides the above-mentioned methods, there are other methods for formulating a variable stiffness design problem, though it may not take into account the continuity of the structure or it may be limited to a narrow class of structures. For instance, for shape and thickness optimization of laminated 2-D structures, Muc [68] presented special types of continuous design variables that represent the locations of several key-points within the laminate and the thickness of the laminate at these key-points. A continuous profile was assumed for the thickness variations from one key point to another. A similar approach was used by Surendranath et al. [101] who used a gradient architecture to formulate the problem. In this method the thickness of the laminate at each point at a given distance from the start of the gradient layer was continuously described by a power law equation (see Fig. 2).

Discrete Material Optimization (DMO, [98] is a formulation that merely deals with parameterization of the fiber orientations. In this method, the mechanical properties of each layer in a composite laminate are computed as a weighted sum of the properties of a finite number of candidate layers each with a different fiber orientation. The design variables are the weighting factors for each candidate layer and the objective is to drive the influence of all but one of these layers to zero at each point. The same concept is used in a technique known as ply-orientation-identity variables [27,21].

Special care must be taken when using different parameterizations as problem convexity and precise definition of the feasible region is strictly connected to the definition of the design variables. While some definition may ease application of continuity constraints, the others may reduce nonlinearity of the problem.

3. Gradient-based methods

This section reviews the application of the traditional gradient-based optimization methods in variable stiffness design of composite laminates. We examine both the algorithms dealing with the objective function and those dealing with an approximate model of the objective and constraints. Although the methods resorting to optimality criteria may also use a traditional gradient-based method, we decided here to include them under another category described in Section 4.

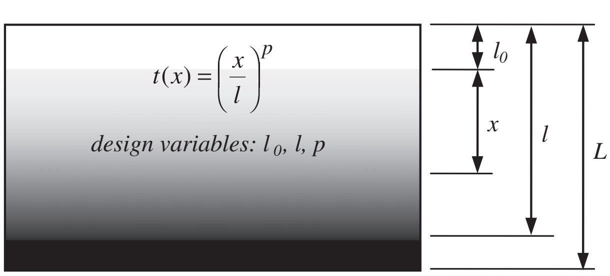

Fig. 2. Thickness of the laminate is described using a one-dimensional power law.

3.1. Methods working with the original problem

The traditional optimization methods generally rely on the sensitivity of the objective with respect to the design variables; thus, they require calculation of the gradient of the objective function and the constraints with respect to all design variables. The effort to calculate the gradients exponentially increases with the number of design variables, thus many authors considered only a limited number of design variables or resorted to analytical methods to calculate the gradients. For instance, Jorgensen [39] used material orientation at each element of a cantilever plate as the only design variables and used an analytical expression derived by Pedersen and Seyranian [80] to calculate the gradients of the objective (i.e. the flutter load) with respect to the design variables. The gradient information was used to make a small change in each design variable such that it causes an improvement in the objective. Cho and Rowlands [14] also used this technique but a finite element method was used to calculate the gradient and the method of feasible directions was in charge of improving the objective. Hyer and Charette [37] and Hyer and Lee [38] used a similar method but reduced the number of design variables by using the patch design. The expected improvement in convergence rate was accompanied by a low blending quality due to a large difference between the fiber orientations in adjacent regions.

Wang and Costin [110] and Costin and Wang [17] introduced ply drop-off constraints and used the method of feasible directions to solve the optimization problem of a wing structure. They used a laminate with four pre-defined fiber orientations and formulated an optimization problem consisting of the percentage of each fiber orientation as a design variable. Constraints were applied to the rate of change in the ratio of the total thicknesses of the two adjacent elements (i.e. ply drop-off rate). To improve the manufacturability of a composite aircraft wing, Gou [26] divided the entire structure into four regions, namely: upper skin, lower skin, front spar and rear spar. He used the method of Davidon–Fletcher–Powell (DFP) to maximize the flutter speed by tailoring only the fiber orientations in these regions.

Even though these methods require the calculation of the gradients, considering their higher convergence rate, they generally require fewer number of iterations than an evolutionary method. However, the nature of the solutions, which are local optima, and the extensive computational time often involved in numerical calculation of the gradients are major drawbacks preventing their wider application in this field. The calculation of the gradients can be particularly challenging when evaluation of the objectives involves time-consuming finite element analyses or costly experiments.

3.2. Approximation methods

Since closed-form expressions of the objectives and constraints often do not exist in a composite design problem, the primary optimization problem might be replaced by a sequence of explicit approximate sub-problems generated through the first or the second order Taylor series expansions of these functions in terms of the design variables [13]. As a result, the sub-problems can be solved by a traditional optimization technique. The optimization method used in PASCO, a computer code developed by Anderson and Stroud [7] and Stroud and Anderson [100], is one of the earliest examples of using an approximation technique in the design of composite laminates. The method was based on Taylor expansions of the objective function and the constraints. Derivatives were calculated numerically by making separate small increments to each design variable. An optimization method based on the feasible direction algorithm was used to solve the approximate problem at each step. The method was tested on simple test cases; however, the extension to the case of a complex structure, which might have a large number of design variables and require a finite element analysis, can be problematic since the evaluation of the derivatives with respect to all the design variables is required. The sequential quadratic programming technique, which uses a quadratic model to approximate the objective function and a linear model of the constraint, is another example of approximation methods being used in variable stiffness design of composite materials [72,12].

Instead of fiber angles and thicknesses, Setoodeh et al. [89,92] used the lamination parameters as design variables. The problem of minimum compliance design of a rectangular plate was solved using a combination of sequential quadratic programming and method of feasible directions. This method relies on the in-plane/bending behaviour of thin, symmetric composite laminates which can be fully modeled with only four lamination parameters. In another attempt, Setoodeh et al. [91] used Taylor expansion for a special case where the approximate function was separable and each of its terms could be expressed only as a function of the fiber-orientation angles at one point. The approximate objective was obtained by expanding the objective function in terms of the components of the compliance tensors. To obtain a smooth distribution of the fiber orientations, the design variables were associated with the nodes rather than the elements.

For simple test cases, an approximation scheme can produce feasible or nearly feasible solutions with a good accuracy; however, the extension of the method to laminates with complex shapes – analysed by finite element method – should require the evaluation of all the derivatives at each design point and may be hard to deal with when the objective function is characterized by steep "peak and valleys", as it is often the case when the fiber angles are assumed as design variables [16]. In addition, the maximum allowable change in the design variables must be defined by the user at each step and it plays a major role in the trade-off between the accuracy and the convergence.

The response surface method [69] is another means of generating an approximate model of a computationally extensive function. In this method, instead of creating a local approximate model based on the value of the function and its gradients at a certain point, an approximate model is generated using a set of sample points over the entire design space. The model is improved by gradually reducing the design space and sampling new points in promising areas. An example is the work by Vandervelde and Milani [107] who used the adaptive response surface method by Wang and Dong [111] to optimize a multi-zone composite wing structure for vibration. A quadratic response surface model was used to find the optimum laminate for each zone. The formulation of the response surface is problem dependent and can strongly affect the accuracy of the approximation. A response surface method can find the global optimum of a multi-modal function, but it may miss the global optimum if the overall shape of the function is convex or if the initial points cause an inappropriate reduction of the design space [111].

4. Optimality criterion

The optimality criteria methods are based on the derivation of an appropriate criterion for specialized design conditions and developing an iterative procedure to find the optimum design [86]. A short history of optimality criteria methods can be found in Logo [59]. In this section the application of these methods in design of composite materials is reviewed, categorizing them into three groups based on the criterion used for optimization.

4.1. Strain energy

Khot et al. [45] were among the first who used an optimality criterion for the design of composite materials. Their optimality condition was an extension of the optimality criterion for isotropic materials [108,109] and was stated as follows: "the optimum design is the one in which the strain energy of each layer bears a constant ratio to its energy capacity". They optimized the thickness distribution of four layers with fibers oriented at 0°, 90°, +45° and −45°. An iterative procedure was proposed that included two steps; the first step aimed at scaling the overall thickness to size the laminate to the limit specified by either a stress or a displacement constraint, the second step entailed the change of the relative thickness of the layers to achieve an even distribution of the strain energy among layers. The method can be incorporated into a finite element approach to optimize practical structures; however, indirect consideration of the fiber orientations can lead to a problem with numerous design variables.

In contrast, Pedersen [74] focused his attention only on the optimization of the fiber orientations. In his work, energy density of a uniform strain field was considered as the optimality criterion to be maximized (for maximum energy absorption) or minimized (for maximum stiffness). The closed-form mathematical expression was developed using the angle between the principal strain coordinate system and the material coordinate system as the only design variable. The optimum material orientation was obtained by performing a sensitivity analysis of the specific elastic energy with respect to the selected design variable. The method was also applied to the case of a variable strain field, where the strains were calculated by a finite element method [75].

In order to minimize the compliance and the price of a composite structure, Duvaut et al. [20] introduced a combined criterion consisted of the work done by the external forces and the material cost (i.e. a function of the fiber volume fraction). Using the stress field found by a finite element analysis, they calculated the strain energy and the cost at each element. The infimum of this function provided the fiber volume fraction and fiber orientations for the next iteration. The method could quickly reach an optimum solution; however, the uniqueness and globality of the solution is unsure. Setoodeh et al. [90] also used the complementary work done by the external loads for the design of a structure with minimum compliance. The optimality criteria were shown to reduce to the minimization of the complementary strain energy at each point in the domain. Lipton and Stuebner [56] used a similar criterion in the method called *inverse homogenization technique*. Khosravi and Sedaghati [44] also used the strain energy, but in a two-level optimization technique. The first level aimed at finding the best fiber orientation by minimizing the strain energy, while the second level searched for the optimum thicknesses to achieve a uniform strain energy through the thickness. The new thicknesses were then scaled to satisfy the stress constraint in the form of a failure criterion.

Optimizing a structure by finding the minimum or maximum strain energy has also been formulated with respect to the lamination parameters. The goal is to change the parameterization and obtain a convex design space which is easier to solve than the original problem. Hammer et al. [32] and Setoodeh et al. [89] used this formulation and considered the minimum compliance energy as optimality criteria. Abdalla and Gürdal [2] also selected the lamination parameters to maximize the natural frequency of composite panels. They developed a discrete optimality criteria based on a generalization of the reciprocal approximation. Generally, using the lamination parameters requires knowing the feasible domain of the lamination parameters which, so far, has been achieved only for relatively simple laminates [64]. Determining the feasible regions of all 12 lamination parameters for a general laminate may require solving several optimization problems [19] and thus it is far more complicated than being incorporated into an optimization problem. In addition, the inverse problem should be solved to find the stacking sequence that corresponds to the optimum lamination parameters found during the optimization process. These are among the main reasons preventing this parameterization from being applied to complex structures with general laminates.

4.2. Co-alignment of principal directions

An interesting observation reported by Pedersen [75] is that with equal principal directions for material, stresses and strains, always extremum energy solutions are obtained. However, an additional non-trivial extremum solution often exists which may be the global optimum solution. Even for this case, it was proved that the directions of the principal stresses and strains are identical, but different from the material principal axes. For a general anisotropic material, Pedersen [77] showed that the optimum material is orthotropic and the co-alignment of the principal strains and stresses still holds. Thomsen [104] and Landriani and Rovati [51], who found the optimum laminate by minimizing the elastic energy, found the same colinearity between stresses and strains in the optimum design; however, they also observed that in some cases, such as angle-ply composites, the colinearity between stress, strain and material direction might be lost. Setoodeh et al. [90] summarized the observations from the previous works as follows: "the fiber orientation for shear-weak materials coincides with the principal stress direction, while for shear-strong materials this may not be valid". For the definition of shear-weak and shear-strong materials, the reader is referred to Pedersen [77].

To design a composite laminate with a given volume, Pedersen [76,77] used the colinearity between the principal strain and material directions to optimize the fiber orientations, while the thickness distribution was optimized using uniform strain energy. An optimality criterion was developed to update the angle between the material orientation and the current principal strain direction in each element. The thicknesses were updated at each iteration by simply multiplying the thickness of each laminate by the ratio of its energy density to the average energy density of the entire structure. The mutual influence of neighbouring elements on the energy density of the current element could also be considered by calculating the corresponding sensitivities. This approach was also applied to non-linear anisotropic materials classified as powerlaw elasticity [78] and was shown that both optimality criteria (i.e. uniform energy density and co-alignment of principal directions of material, strain and stress) are valid. Pedersen [79] showed that the optimum design found by this method corresponds to the largest stiffness and the strongest design only when there is a constraint applied on the total volume.

The colinearity between principal stress, strain and material directions was partially used in the fiber steering technique, which aims at aligning the fibers with the principal stress directions at each point of the structure. It was first used by Hyer and Charette [37] to design a plate with a hole uniformly loaded at its two ends. They used a stacking arrangement denoted as $[\pm 45/\theta]_s$, where θ showed the only layer in the laminate with variable fiber orientation. The fibers were initially aligned with the principal stress directions for an isotropic plate. At each iteration, the fibers were realigned with the new principal stress directions. The laminate found by this method showed an improved tensile strength; however, the buckling loads were generally less than the baseline quasi-isotropic design. The code called computer aided internal optimization (CAIO) developed by Kriechbaum et al. [48] also used the alignment of the fibers with the principal stress direction. Crothers et al. [18] who used this code to design the local reinforcement of a notched plate reported that this method do not yield an optimum structure when the buckling loads are critical. There are other examples where the fiber steering technique is used to improve stiffness [106] and strength [54,55] of composite structures.

Tosh and Kelly [105] showed that in some cases adding fibers in the direction orthogonal to the first principal stress direction can improve the strength; therefore they suggested to place a small number of plies in the orthogonal direction besides those plies already aligned with the first principal stress direction. A decomposition of combined loadings into the constituent single-loads was also suggested with the goal of aligning the fiber orientations at each point to the major principal stress directions obtained for each single load case. Successful application of the superposition technique was demonstrated only on a pin-loaded plate with tension applied at the two ends.

To achieve the maximum strength, Tosh and Kelly [105] suggested steering the fibers to the direction of the load paths instead of aligning them with the principal stress directions. The load paths were defined as regions where the load in a selected direction remains constant from the point of application to the point of reaction out of the structure. Kelly et al. [42,43] described a procedure for plotting load paths and load flow in structures using the stresses found by a finite element analysis. The load paths and fiber steering techniques, which was employed in the design of an open hole tension plate, showed similar results, the major difference was that no fiber was terminated on the boundary of the hole in the load path configurations.

4.3. Other criteria

The well known *Fully-Stressed-Design* (FSD) is an optimality criterion widely used in the design of isotropic materials. This criterion states that every component in the optimal structure should be on the verge of the failure for at least one of the applied loads [15]. The optimality criterion is easy to implement for structures made of isotropic materials by using a stress rationing algorithm; however, its application to composite materials is more complex as it requires optimization of the thickness and the fiber orientation. Kurland [50] used an educated trial and error to find the fiber orientation and thicknesses for the next iteration of the FSD method. This combination of thickness and fiber orientation optimization was developed into a computer code called Hybrid Algorithm for Laminate Optimization (HALO) whose application to several test cases was demonstrated by Fine and Springer [22]. The attempt by Leissa and Vagins [53], who designed a non-homogeneous material for yielding a known optimum internal stress field (e.g. a uniform stress field), can also be categorized in this group. It has been shown [24] that a structure optimized with the FSD method may fail to be the lightest design. For composite structures, the scenario becomes even more inefficient as either several iterations are required or the method might never converge.

Besides the criteria described above, there are other optimality criteria developed for special cases. Examples are the momentless criteria by Pao [71] and a combined criterion by Pedersen [73]. The momentless criterion was derived for axisymmetrically loaded pressure vessels with ellipsoidal bulkheads using the volumetric ratio of the fibers at each point as design variables. The combined criterion, which was a linear combination of the bending stiffnesses was proposed for a special case of orthotropic laminates. It was shown to be proportional to the inverse of the displacements, the buckling load and the square of natural frequencies. The functional was derived for a rectangular plate and was reported to be very sensitive to the displacement pattern (i.e. vibration or buckling modes). The optimum fiber orientation and thickness at each element was found by simply setting the gradient of the optimality criterion to zero. Thus, being unconstrained and having a stationary point was a necessary condition for the optimization problem. This assumption was found to be incorrect for many cases where the thickness optimization was involved.

5. Topology optimization

Topology optimization finds the optimum distribution of material within a prescribed design domain for a given set of loads and boundary conditions. The optimal layout results from an iterative process which involves the gradual removal of material in the regions where the material is redundant and adding material in the areas of the structure where it is required. When the objective is a local state variable, the problem well suits an optimality criterion method, such applications are getting a growing attention from designers in different fields. On the other hand, when the objective is a global state variable it can be solved by variety of other optimization methods but it requires the sensitivity analysis of the objective with respect to each design variable. Such applications have not been found in the current literature. Generally, topology optimization has been only recently applied to the design of composite structures and further research is required to improve its performance and increase its applications. This section briefly reviews the development and application of topology optimization methods for design of composite structures.

Ignoring the fiber orientation simplifies the composite design problem and enables classical topology methods to find the material distribution. Since the advantages of the composite materials cannot be fully put into practice without consideration of material orientation, examples using only density as design variable are rare. One of such applications is reported by Tapp et al. [102] who designed a sandwich structure in which the fiber orientation of the face sheets was kept constant for easy manufacturing. Topology optimization of both the face sheets and the core was obtained by adding or subtracting material from regions of high or low stress, respectively. Stegmann and Lund [99] used a similar approach but resorted to *Solid Isotropic Material Penalization* (SIMP) to generate a structure with only solid or void layers.

In contrast to the first group, there are methods dealing exclusively with fiber orientations. Discrete material optimization (DMO, [64] described in Section 2.4, is one of these methods in which the shape and thickness of the structure are fixed and the problem merely deals with parameterization of the fiber orientations. These methods are based on ideas from multiphase topology optimization in the sense that the material stiffness (or density) is computed as a weighted sum of stiffness (or density) of candidate materials.

Simultaneous consideration of material distribution and orientation is essential for designing an optimum composite structure. One simple approach is to optimize the density using a classical topology method, while the fiber orientation is aligned with the first principal stress direction (fiber steering technique). Ma et al. [65] used this technique, where the material distribution was optimized using *multi-domain topology optimization* (MDTO) and the fiber orientations were determined by using either the principal stress directions or a proposed analytical equation which is updated at each iteration. MDTO is an extended form of the classical topology optimization called *homogenization based topology optimization* (HBTO, [11]). Fuchs et al. [23] also considered the alignment of the fibers with the first principal stress direction, but the material distribution was obtained by driving the densities to zero in under-stressed regions and to one in the remaining ones.

Another method that accounts for both the density and fiber orientation is the layerwise topology optimization suggested by Hansel and Becker [33]. This method starts with a symmetric laminate composed of four single layers with equal thickness and with the fibers oriented at 0°, ±45°, and 90°. At each point, a single layer whose fiber angle differs significantly from the principal stress directions is removed. The next step updates the stress field and removes the cells whose maximum principal stress is below a user-defined threshold. In another work, Hansel et al. [34] used the same formulation but a genetic algorithm was introduced to remove unnecessary layers and cells. To partially overcome the numerous analyses required by a genetic algorithm, some additional criteria for removal and supplement of material was included. In this case, the higher computational cost of the genetic algorithm did not lead to a remarkable improvement of the results.

Similar to two previous works, Zhou and Li [118] also formulated a topology optimization problem which used different criteria to update the fiber orientations and densities. They built a *truss-like continua* (see Prager and Rozvany [82] for definition of the truss-like continuum) by finite element method. At each iteration, the fiber orientations were updated by solving the problem of minimum compliance, while fiber densities were updated by a resizing scheme based on stress and strain energy. At the end of the process, they transferred the truss-like continua to discrete structures. This process is similar to forming streamline in a flow field. Errors might emerge in this process, but Prager [81] and Zhou and Li [116] showed that these errors would decrease rapidly as the number of the members increases. Finally, the shape optimization of trusses was performed by eliminating overly thin members. By the same token, this method can be used to determine the fiber orientation distribution within a composite structure [117].

The use of different optimality criteria to find the optimum density and fiber orientation generally simplifies the optimization problem; however, such criteria may not be easy to find and, if not properly defined, may misjudge the effect of each variable on the final objective. Setoodeh et al. [90] used a single criterion for optimizing fiber orientations and layer thicknesses by expressing the strain energy density of the structure in terms of fiber orientations and fictitious densities. The minimum strain energy problem was then converted to a minimization of the complementary strain energy at each element. First, using the gradient information, the fiber angle was updated to achieve a lower value of the strain energy. Then, using this fiber angle, the density was optimized to obtain the minimum complementary strain energy. *Solid Isotropic Material Penalization* (SIMP) was used to generate a *black-and-white* design almost free of any gray areas (i.e. material or void only) and the *Cellular Automata* was used to reduce the computational effort.

6. Direct search methods

Designing a composite structure with a variable stacking sequence often develops into a global optimization problem with a mix of continuous and integer variables where usually the sensitivities are extremely difficult to compute. Direct search methods are particularly useful for these problems, because they opt out the use of any gradient information; however, they may present other shortcomings, as discussed in this section. Here these methods are classified into deterministic and stochastic methods.

6.1. Deterministic methods

A deterministic direct search method (e.g. pattern search, method of Hooke and Jeeves, Nelder–Mead method) may fail to obtain the global solution when the corresponding optimization problem has several local optima. In addition, these methods are generally not efficient in solving problems with a large number of design variables (e.g. more than ten variables). Such circumstances often occur when the fiber orientations in a composite structure are selected as design variables. Therefore, deterministic methods are not popular in variable stiffness design of composite laminates. Examples of deterministic direct search methods being used for variables stiffness design are Nelder–Mead (N–M) simplex method [66] and Box's complex method [47].

Manne and Tsai [66] resorted to the ply-drop and sub-laminate concept to formulate the variable stiffness design problem. The design variables included the fiber angles of the plies in the reference sub-laminate, the number of plies at the corresponding angle, and the number of times the sub-laminate is repeated in each zone. The stacking sequence of the reference sub-laminate was kept constant for all the design zones, whereas the number of repetition of the sub-laminate could vary from one zone to another. The objective was minimum weight with constraints on stiffness, strength and manufacturing cost. The problem was solved using N–M method in conjunction with a finite element solution. Manne and Tsai commented on shortcoming of the N–M method in getting trapped in local optimum and confirmed that numerous local optima exist in the considered design problem. Kim et al. [47] used the Box's complex method to optimize only the thickness distribution of a composite structure manufactured by compression molding process. This method finds a local optimum by iterative use of two operators, "expansion" and "contraction", acting on a simplex of size $2N$ (as oppose to Nelder–Mead that uses "reflection", "expansion", "contraction" and "shrink" and operates on a simplex of size $N + 1$), where N is the number of design variables. The two methods are similar in nature and highly sensitive to the number of design variables.

6.2. Stochastic methods

Stochastic optimization algorithms, such as genetic algorithm, evolutionary programming and simulated annealing, are popular due to their capability to find a global optimum and their robustness with respect to the deterministic approaches; however, they are generally computationally intensive [113]. Among numerous stochastic optimization algorithms, genetic algorithms are the most commonly used in variable stiffness design of composite laminates.

Legrand et al. [52] used a genetic algorithm to find the optimum orientation of fibres at each element of a composite plate. The fibre trajectories were then expressed through the Runge–Kutta algorithm. The attempt was judged successful as it could interpret a 2D continuous field for the fiber path. This method requires a large number of elements in order to represent the continuous changes in the fiber angles and is applicable only to single-layer laminates. For multi-layer laminates, Antonio [8] used a hierarchical genetic algorithm and considered a single stacking sequence for each panel of a stiffened structure. The stacking sequence of each panel and the stiffener geometry were chosen as independent design variables during the optimization. Although easy to implement, an independent design of the stacking sequences in adjacent panels usually results in a non-blended structure, which may not only increase the lay up cost, but also be structurally unsafe due to discontinuities.

Kim et al. [46] are among the first who targeted the blending issue in composite laminates designed with a genetic algorithm. They applied a genetic algorithm to minimum weight design of a composite structure divided into several patches. In their method, the minimum number of layers at each patch was determined by means of an artificial ply, a super-ply which has the best property of a composite laminate in all directions. The artificial plies were then replaced by real plies and the fiber orientations of the resulted laminates were optimized with a genetic algorithm. In order to satisfy the continuity of the fibers, the fiber orientation of all patches were adjusted with the patch with the maximum number of layers. A ply was added to any patch that did not satisfy the strength requirement and the genetic algorithm was repeated to find the optimum fiber orientations. A good level of blending can be achieved with this method, but the optimality of the final solution is questionable since the most critical patch determines the lamination sequence of the entire structure.

To partially account for the contribution of the non-critical points in determining the fiber orientations within the structure, Soremekun et al. [97] introduced a two-step methodology based on the sub-laminate concept. First, the individual panels in any two-dimensional array of laminated composite panels were optimized separately. Then sub-laminates with common thickness zones across multiple panels were identified and re-optimized by using blended stacking sequences. The identification of the sub-laminates was performed by the user and could be trained to reach a high-performance design or a design with better blending properties. A similar approach was adopted by Zehnder and Ermanni [114], who modelled the structure as an assembly of several overlapping patches, where genetic algorithm was used to define the material and the fiber orientation at each patch. Their method relied on the user to define the patches' geometry, which can be considered as a major drawback of this method. In another work, Zehnder and Ermanni [115] studied the optimum shape and placement of a single patch placed on a shell structure made of one base material. The patch design introduced is far underdeveloped to be efficiently used for breaking down even a simple structure to an assembly of overlapping patches. In addition, the optimality of the solution strongly depends on the quality of the pre-defined assembly.

Blending constraints are also used to achieve better blending properties in a design found by genetic algorithm. Liu and Haftka [58] proposed to apply constraints on a measure of the material composition continuity and the stacking sequence continuity between two adjacent panels. The continuity was measured by the ratio of the number of continuous layers to the total number of layers. Their optimization method included two levels; at the global level the material composition of each element was determined and at the local level a genetic algorithm was used to find the stacking sequence of the plies. The constraints could be adjusted to provide solutions with different level of continuity. It was demonstrated that a substantial improvement in continuity could be achieved with a minor weight penalty when the method was applied to the design of a composite wing structure; however, in this particular test case, the small number of allowable fiber angles ruled out the use of the stacking sequence continuity constraints. In a more general problem, enforcing the continuity constraints may create a highly constrained problem, which can be very difficult to solve with a genetic algorithm.

Some authors suggested modifying the genetic algorithm itself to achieve a continuous structure without any additional constraints. The multiple elitist genetic algorithm of Soremekun et al. [96] has been considered by Adams et al. [3] as one of these attempts that can help achieving a globally blended solution by providing a large spectrum of alternative elite designs; however, the work was originally developed for constant stiffness design and neither aimed at nor could guarantee achieving a globally blended design.

McMahon and Watson [67] proposed a parallel genetic algorithm with migration where the populations representing each panel of a multi-panel structure evolve in parallel, and periodically send migrant individuals to adjacent populations. The migrating individuals transmit the information between the adjacent panels and increase the similarity between the lamination sequences of two adjacent panels. Adams et al. [3] slightly changed the migration procedure where the migrant individuals were stored and were referred to as needed to assess the compatibility of the current evolutionary trends with neighbouring populations. The individuals that were evolving closely (the similarity was measured by a metric called *edit distance*) to those in the migrant population from adjacent panels were rewarded by an increase in their fitness value. A limited success was achieved in designing a completely blended overall structure and the results were reported to include structures with varying degree of blending property. Evolutionary pressures were controlled through a user defined scaling factor that modified the severity of the penalties imposed for blending mismatches. These penalties, however, were found to hinder the convergence to a global optimum by creating local optima, which are artifacts of the algorithm itself, in the search space [4]. Also, increasing the number of migrants had an adverse effect and caused the GA to converge to locally non-optimal laminates with sometimes severe mismatch between adjacent panels.

Rather than performing several parallel genetic algorithms, which can be computationally cumbersome, Adams et al. [4] proposed a guide-based genetic algorithm. This method assumes that the lamination sequences of all the panels can be obtained from a guide laminate by determining the number of the contiguous inner or outer plies to be removed from the guide laminate. A GA was in charge of generating a population of lamination sequences that guided the overall design process. This formulation reduces the dimensionality of the problem and eliminates the need for continuity constraints; however, the simplified definition of blending used in this method entails the loss of flexibility to trade the degree of blending against weight. It was reported that this method consistently produced better solutions than the parallel GA [3]. Adams et al. [4] assumed the individual panel loads to be constant during the design process. This shortcoming was later targeted by Adams et al. [5] and Seresta et al. [88], who expanded the method for the case where the local loads for individual panels were determined through a global-level analysis. The global/local analysis was iterated until the convergence was achieved. The stacking sequence continuity achieved by the guide-based genetic algorithm was reported higher than the one achieved by using the continuity constraints [58].

Tatting and Gürdal [103] used a three parameter curvilinear fibre path definition to model the fiber orientation in variable-stiffness laminates. A genetic algorithm was used to optimize the fiber path for maximum buckling load. A similar trend was used by Huang and Haftka [36], who resorted to a piecewise bilinear interpolation function to represent the distribution of the fibers near a hole in each layer of a multilayer composite laminate. The optimization process entailed a combination of conjugate gradient method and a genetic algorithm. The use of a parametric fiber path significantly reduces the number of design variables as well as it guarantees the continuity in the fiber path of each layer; however, the quality of the final solution strongly depends on the selected parametric function, whose definition is not a straightforward task for a structure with a complex geometry.

7. Multi-level optimization

Solving an optimization problem that includes all the variables describing a complex structure is impractical. Multilevel approaches are capable of breaking down the optimization problem into several optimization problems that can be solved separately in an iterative process. A hierarchical decomposition divides the problem into a system level problem and a set of uncoupled component level problems. Such hierarchical decomposition has been used in the design of metallic and composite structures [94,10,30]. There are also non-hierarchical decompositions that divide the problem into several parallel problems. The most common form of this decomposition applied to composite materials consists of decoupling the optimization of the thicknesses from that of the fiber orientations [35]. At one level, only the thickness is optimized, leaving the search for the best fiber orientation for each layer to the second level. Application of the two mentioned types of decomposition in variable stiffness design of composite materials is studied in this section.

7.1. Hierarchical substructuring

A hierarchical decomposition of the design problem has been widely used for the design of complex systems made of traditional metallic materials. Schmit and Mehrinfar [87] were the first who extended and applied this method to the design of a hat-stiffened composite panel. They divided the design variables into system level and component level variables. At the system level, the weight of the structure was minimized subject to strength, deflection and overall buckling constraints. At this level, the hat-stiffened structure was modelled with an equivalent laminate, where the thickness of its four layers (i.e. with fibers at 0°, 90° and ±45°) were the only design variables. At the component level, the detailed design variables were obtained by minimizing the change in the stiffness of the skin panels subject to a set of local buckling constraints. The optimization procedure employed for both system and component levels was based on the quadratic extended interior penalty function which led to a sequence of unconstrained minimization problems, each solved with the modified Newton method. In their work, the design variables were only the thickness of the lamina with predetermined fiber angles. In order to weaken the coupling between the two levels and to ensure convergence of the overall design, an appropriate decomposition guided by insight into the physics of the problem and a proper modeling approach are necessary. For simple structural models, the integration of the two levels can be handled well; however, for complex configurations, finding an effective decomposition can be cumbersome and the resulting solution may be suboptimal.

Liu et al. [57] minimized the weight of a wing structure using continuous optimization of thicknesses of four plies with orientations of 0°, 90°, and ±45°, at the upper level. At the lower level, the number of plies of each orientation (rounded off to an integer number) was specified and a permutation genetic algorithm was used to optimize the stacking sequence for buckling load maximization. A response surface trained by performing hundreds of panel optimizations was used to estimate the optimal panel buckling load during the upper-level optimization. The use of a response surface method in multilevel approach is also reported by Ragon et al. [83,84]. In their earlier work [84], the optimization was performed with the same objective function (structural weight) at both wing and panel level. Using the same objective at two levels could yield situations where no feasible solution exists for the lower level problem. To solve this problem, Haftka et al. proposed using a maximum margin formulation at the lower level. That is, at the panel level the objective of minimizing weight was changed to that of maximizing the constraint margin; if no feasible design was available, then the algorithm could search the design with the smallest constraint violation [31].

Another approximation scheme was suggested by Sobieszczanski-Sobieski et al. [94] who proposed calculating the sensitivity of the derivatives of the lower-level optima and using them during the upper-level optimization. This method has not been used for composite materials probably due to problem of working with the functions which are highly non-linear with respect to the system level design variables.

Hierarchical decomposition may also be used to break down the system to several component-level optimization problem linked together through a global-level analysis. In this case, no optimization is performed at the system-level, but the system-level analysis is used only to update the information at the component level. For instance, Duvaut et al. [20] proposed a decomposition where the component-level optimization resorted to a stress field calculated by a global-level analysis. The method by Adams et al. [5] also used the local loads for each panel determined through a global-level analysis. The method was claimed to converge quickly; however, the uniqueness of the obtained solution and its optimality for being a global or a local optimum cannot be proven.

7.2. Non-hierarchical decomposition

One of the earliest applications of a non-hierarchical decomposition in design of composite material is reported by Watkins and Morris [112]. In this method, at the upper level the optimization was applied to the entire structure, where the layer thicknesses were the only design variables. At the lower level, the fiber orientation at each element was considered as design variables and the problem was to minimize the weight while constraining the change in stiffness of the element to a minimum. This ensured that the stiffness, and hence the load paths, within the overall structure do not change substantially, thus the continuity was preserved when the solution returned to the upper-level optimization.

Kam and Lai [41] and Kam and Chang [40] used a similar decomposition, where the upper level consisted of finding the fiber orientation for each layer of a multi-layer laminate that maximizes the material efficiency, such as stiffness, strength, natural frequency or damping. While a quasi-Newton method was used at the first level, the second level used an optimality criterion to scale the thickness of each layer to the constraint surface. A similar decomposition was used by Huang and Kroplin [35] to design a laminate whose fiber orientation was specified by only one variable, thus all the derivatives required for the first level were analytically calculated. All these works reported a good convergence property and independency on the constraint's nature (stress or displacement). Soeiro et al. [95] and Antonio et al. [9] suggested using a conjugate gradient method at the first level and calculating only the maximum constraint sensitivity at each iteration. The proposed method has a better computational efficiency and was applied to a multi-element structure. Khosravi and Sedaghati [44] used this method to find the fiber angle and thickness at each element of a complex structure analysed by a finite element method.

In addition to the common decomposition of the problem into fiber angle optimization and thickness optimization, other types of decompositions have been proposed for some particular test cases. To design a balanced-angle-ply-symmetric laminates with membrane loads Conti et al. [16] suggested finding some elastic properties and the corresponding lamination sequence, separately. They proposed to search the optimum value for A_{11} and A_{22} (the first two diagonal elements in the stiffness matrix), assuming that the feasible domain for a laminate with limited possible fiber orientations and thicknesses is known. It was then shown that any pair of elastic characteristics can be obtained with maximum three orientations. Thus, the second level was formulated to choose the angle triplets which can yield these engineering characteristics. A graphical method, called *space partitioning*, was proposed to solve this problem. Another decomposition is reported by Lopes et al. [60,61] and is based on the observation that the impact footprint on the alternative laminate is narrower than on the traditional design with fibers oriented only at 0°, 90°, ±45°. Their method entailed two steps. First, it searched the traditional design that provides the best stiffness properties. Second, redesigning the obtained laminate to better withstand impact loads by dispersing its stacking sequence while keeping similar stiffness properties. Another example of possible decompositions is separating the optimization of the number and the position of the layers within a laminate subjected to in-plane loads.

There are two advantages in using decomposition techniques. First, by employing a decomposition technique and a multilevel algorithm the number of variables is reduced during the optimization process, which can significantly reduce the computational effort required. Second, by performing the optimization at two separate levels, one can choose an appropriate optimization method that is particularly efficient for each sub-problem and benefits from advantages of two or more optimization methods. The implementation of decomposition technique also suffers from two main shortcomings. First, decomposition approaches require physical insight into the problem and needs establishing a close integration of the tow levels, which has prevented their ready application with the black-box codes often used in the aircraft industry [83]. Second, the efficiency of the method (i.e. convergence) and the quality of the solution (i.e. optimum or near-optimum) strongly relies on the decomposition; for instance, it is recognized that a minimum weight structure is not necessarily made up of a collection of minimum weight substructures [93]. Furthermore, difficulties may arouse in convergence when the lower level problems are described with highly non-linear functions in terms of design variables.

8. Hybrid methods

A hybrid method refers to an optimization algorithm that uses more than one optimization technique without decomposing the original problem into sub-problems. A hybrid method usually iteratively switches between two or more optimization methods in order to benefit from the advantages of each constituent method. Although this type of optimization technique is found promising for constant stiffness design of composite laminates [25], their application in variables stiffness design is dominated by multi-level methods, which besides having the potential of gaining benefits from more than one optimization technique, can also reduce the size of the problem.

Huang and Haftka [36] combined the Conjugate Gradient (*F–R*) and a genetic algorithm to avoid local optima in the design of the fiber orientations near an open hole in a multilayer composite laminate. The only design variable was the fiber angle in one layer of the multi-layer laminate and was assumed to have a continuous distribution represented by piecewise bilinear interpolation functions. The algorithm consisted of optimizing the failure load using conjugate gradient (a local optimum) and performing a genetic algorithm on a population generated by random mutation of the local optimum. The objective of the genetic algorithm was to move the optimization to a better region, while the conjugate gradient method was used to quickly find the local optimum in this region. Rao [85] used a similar combination but used *ant colony optimization* (ACO) for the global search, while a fast local search was performed by a neighbourhood search algorithm built with tabu search features.

For the particular case study of the in-plane/bending behaviour of a thin symmetric composite laminate (which can be fully modeled using only four lamination parameters regardless of the actual number of layers) Setoodeh et al. [89,92] solved the minimum compliance problem by combining the sequential quadratic programming and the method of feasible directions. This mix of directions was adjusted to ensure feasibility while retaining fast local convergence properties.

9. Comparison and discussion

Table 1 compares the optimization methods reviewed in this paper. Three major features were considered in this comparison, namely, convergence, robustness and simplicity. Convergence is

Table 1
Comparison of the optimization methods used in variables stiffness design of a composite structure.

Method		Convergence	Robustness	Simplicity
Gradient-based	Incremental move in gradient direction	***	*	*
	Davidon–Fletcher–Powell (DFP)			
	Method of feasible directions			
	Linear approximation	**	*	**
	Sequential quadratic programming			
	Response surface method	**	*	*
Optimality criterion	Strain energy	**	*	**
	Fiber steering (aligning the fibers with first principal stress direction or load path)	*	*	**
	Fully stressed design (FSD)	**	*	**
	Combined criteria	*	*	*
Topology	Thickness-only optimization	**	*	*
	Thickness and orientation optimization with a local objective	**	*	*
Direct search	Nelder–Mead	**	*	***
	Box's complex methods			
	Genetic algorithms	*	***	***
Multi-level	Hierarchical decomposition (system and component level)	***	*	*
	Non-hierarchical decomposition (thickness and orientation)			
Hybrid	Conjugate gradient + genetic algorithm	**	**	*
	Sequential quadratic programming + method of feasible direction			

* Poor.
** Moderate.
*** Good.

related to the number of iterations required and the amount of computation involved at each iteration. Robustness shows the algorithm's applicability to a wide range of optimization problems. For instance, an algorithm that can be easily applied to a black-box problem is more robust than the one that requires an insight into the physics of the problem. This measure also shows the algorithm's ability in getting close to the real optimum and in finding the global solution of a given multimodal problem. Simplicity in application judges the ease of computer programming and not requiring the interaction with the user. For instance an algorithm that requires fewer user-defined parameters than another is a simpler algorithm. A summary of the advantages and disadvantages of each method is presented in Table 2.

This review revealed that two similar approaches, namely the optimality criterion methods and topology optimization with a local update rule, have attracted the most attention from the researchers dealing with variable stiffness design of composite materials. These methods break down the complex design problem with numerous design variables into a set of simple local problems with a simple updating rule. A major limitation of these methods is the possibility of establishing a local criterion whose optimum corresponds to the optimum solution of the original design problem. Minimum weight, maximum stiffness, maximum energy absorption and minimum material cost are the objectives that can be easily casted in this formulation; in contrast, problems involving optimization of a global state variable, such as buckling, natural frequencies, overall displacement and manufacturing cost are not appropriate for these methods.

Multi-level optimizations are ranked next and well suit a variable stiffness design problem because of decomposing the original problem into several smaller problems. The decomposition, which must efficiently maintain the interaction between the sub-problems, is the key to the optimality of the final solution. The other advantage of these methods is the chance of taking advantage of more than one optimization methods at each level.

Although most direct search methods are not favourable for variable stiffness design due to their low tolerance to a growing number of design variables; genetic algorithms have well proven their ability to handle this type of problems. Despite their simplicity, which is the key to their popularity, slow convergence and difficulties in maintaining the continuity of the structure are the main drawbacks of these algorithms that withhold their wide application for variable stiffness design of composites. Finally, gradient-based methods and hybrid methods, which also usually derive benefit from one of the gradient-based methods for a fast convergence to a local solution, are found to be less favourable in variable stiffness design mainly due to difficulties in evaluation of derivatives with respect to numerous design variables.

10. Concluding remarks

This paper along with its first part, previously published in *Composite Structures* [25] provides a thorough review of the optimization methods used in the design of structures made of laminated composite materials. This part of the review particularly focused on variable stiffness design of composite laminates. The aim was twofold; first, to provide a reference to composite designers willing to select the optimization technique that can best solve a given stacking sequence design problem, and second, to highlight promising areas that require further attention and deserve future research.

In variable stiffness structures, the arrangement of the constituent materials varies from one region within the structure to another and thus numerous design variables are required to represent the spatial variation in stacking sequence. Different formulations used to achieve the continuity in the structure were reviewed, among which the patch design was found to be the most commonly used technique due to simplicity in formulation; however, a proper definition of the patches, which must be done by the user, is important and requires expertise and prior knowledge about the problem.

After reviewing several optimization methods, the methods used in variable stiffness design of composite materials are ranked as follows:

(1) If applicable, the optimality criterion methods and topology optimization with a local update rule are the best candidates for variable stiffness designs. These methods break down the complex design problem into a set of simple local problems with a simple updating rule.

Table 2
Advantages and disadvantages of the optimization methods applied to the variables stiffness design of a composite structure.

Methods		Advantages	Disadvantages
Gradient-based	Incremental move in gradient direction Davidon–Fletcher–Powell (DFP) Method of feasible directions	Converges fast	Finds only a local optimum Requires evaluation of derivatives Deals with continuous variables
	Linear approximation	Can quickly find an optimum or a near optimum solution for simple problems	Finds only a local optimum
	Sequential quadratic programming	Can be solved by a classical optimization method available as commercial computer codes	Requires evaluation of derivatives May converge to a non-optimum solution for a highly non-linear problem
	Response surface method	Requires no derivative	Response surface function is problem dependent
		Can find the global optimum	May miss the global optimum The final solution may depend on the initial points
Optimality criterion	Strain energy	Introduces a local criterion that can be optimized at each point independently Can optimize both material direction and material distribution Can be applied to multi-layer laminates	Can optimize only the objectives related to minimum or maximum energy (e.g. weight, compliance, stiffness, energy absorption)
	Fiber steering (aligning the fibers with first principal stress direction or load path)	Introduces a local criterion that can be optimized at each point independently, simple updating rule	Can find only the largest stiffness and the strongest laminate May yield a non-optimum solution Can optimize only material direction
	Fully stressed design (FSD)	Introduces a local criterion that can be optimized at each point independently Simple in concept	Can find only the lightest structure May yield a non-optimal solution Is difficult to simultaneously update thickness and fiber orientation
	Combined criteria	Introduces a local criterion that can be optimized at each point independently, Can optimize more than one objective	Problem dependent Requires defining appropriate criteria for each problem and objective
Topology	Thickness-only optimization	Simple formulation Simple updating rule	Does not take advantage of the fiber orientation
	Thickness and orientation optimization with a local objective	Introduces a simple independent optimization problem at each point of the structure	Optimality criterion and updating rule are highly problem dependent, Can find only a local optimum Only for single-layer laminates
Direct search	Nelder–Mead Box's complex methods	Does not require gradient information, Relatively fast in convergence (faster than evolutionary but slower than gradient-based methods)	Finds only a local optimum Limited to small number of design variables (approx. 10 variables) Difficult to maintain continuity of the structure
	Genetic algorithms	Does not require gradient information, Can find the global optimum, Can optimized large number of design variables, Can be applied to a black-box problem with any type of objective	Slow convergence Difficult to maintain continuity of the structure
Multi-level	Hierarchical decomposition (system and component level) Non-hierarchical decomposition (thickness and orientation)	Breaks down the problem into simpler problems Can benefit from more than one optimization method Fast convergence	Its performance highly depends on decomposition, May fail to reach optimum
Hybrid	Conjugate gradient + genetic algorithm	Can benefit from advantages of more than one optimization method,	Slow convergence (faster than a pure evolutionary method but can be slower than almost all other methods)
	Sequential quadratic programming + method of feasible direction	Can find the global optimum, Can be faster and/or more robust than the constituent methods	

(2) When such optimality criteria cannot be easily established, multi-level optimization methods are the best candidate. These methods decompose the original problem into several sub-problems, each with a smaller number of design variables. The possibility of such decomposition is the major concern in using these methods; however, composite design problems have been demonstrated to provide a good possibility for such decompositions in different situations.
(3) In problems where neither an optimality criterion nor a multi-level method can be used, depending on availability of the gradients a genetic algorithm or a gradient-based method or a combination of these two methods is recommended.

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