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Review

Optimum stacking sequence design of composite materials Part I: Constant stiffness design

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ABSTRACT

Designing an optimized composite laminate requires finding the minimum number of layers, and the best fiber orientation and thickness for each layer. To date, several optimization methods have been introduced to solve this challenging problem, which is often non-linear, non-convex, multimodal, and multi-dimensional, and might be expressed by both discrete and continuous variables. These optimization techniques can be studied in two parts: constant stiffness design and variable stiffness designs. This paper concentrates on the first part, which deals with composite laminates with uniform stacking sequence through their entire structure. The main optimization methods in this class are described, their characteristic features are contrasted, and the potential areas requiring more investigation are highlighted.

Contents

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1. Introduction

The advantage of composite materials is that they provide excellent mechanical properties. However, using this advantage requires the optimization of shape and size and the proper placement of fibers within the material, which gives a good opportunity to tailor the material properties; however, it increases the complexity of the design problem. This complexity exists, not only because of numerous design variables, but also because of having a multimodal and variable-dimensional optimization problem with unattainable or costly derivatives.

This paper classifies and compares optimization techniques used for optimal lay-up selection of laminated composite materials. The aim of the review is to provide a reference for the selection of the technique that is most appropriate to solve a given problem. More details about the algorithms are found in Haftka and Gurdal [1], Gurdal et al. [2], and other references provided in the following sections.

In the literature, different classifications for optimization of composite laminate have been suggested. For example, Fang and Springer [3] identified four categories for the optimization methods, namely: (1) analytical procedures, (2) enumeration methods, (3) heuristic schemes, and (4) non-linear programming. Abrate [4], on the other hand, decided to classify optimization problems with respect to their objective functions that can be either one or a combination of in-plane properties, flexural rigidity, buckling load, natural frequency, and thermal effects. Venkataraman and Haftka [5] suggested another classification for composite panels, which categorized the design methods into two groups: (1) single laminate design, and (2) stiffened plate design.

We follow the classification provided by Setoodeh et al. [6], in which there are two scenarios for the design of a composite structure:

(I) Constant stiffness, in which the composite part is considered as a single element with the same stacking sequence all over the domain. The design goal is to find an optimal stacking sequence that is uniform for the entire structure.
(II) Variable stiffness, in which the structure consists of multiple elements, each of them with a different stacking sequence. Here, material distribution and fiber orientation might change over the structural domain.

For the first scenario, which is the focus of this paper, we examine the following optimization methods:

1. Gradient-based. These methods utilize gradient information of the objective(s) and the constraint(s) to find the direction and size of the step towards the optimum solution.
2. Direct Search and Heuristic. In contrast to the previous group, these methods do not need any gradient information; rather they require only the values of the objective function. Heuristic and Enumeration methods are also included here.
3. Specialized techniques. These methods are developed for solving a lay-up design problem, in which some properties of composite laminates are used to simplify the problem; thus, they cannot be applied to a general optimization problem.
4. Hybrid methods. In this class fall methods that combine two or more optimization techniques to benefit from the strength of all constituent techniques.

In this paper, we review optimization techniques used for constant stiffness design of composite laminates. The scenario of variable stiffness design will be the focus of a future paper, which will also analyze other issues, such as multi-objective optimization, discretization techniques, design for manufacturing, sensitivity analysis, and design for uncertainty. The following sections describe the main optimization methods for constant stiffness design with respect to the four classes given above. This design scenario is simpler than that for the variable stiffness design, since generally there are fewer design variables involved.

2. Gradient-based methods

These algorithms are based on the gradient of the objective and the constraints, whose functions, when their mathematical closed-form expression is not available, can be approximated, although it may be computationally expensive. The solutions obtained with gradient-based methods are only local optima, but the advantage of these methods is the faster convergence rate as opposed to that achieved by direct and heuristic methods.

2.1. Vanishing the function's first gradient

Setting the first gradient of the objective function to zero is the simplest and the most common method to find a stationary point of a mathematical function. Sandhu [7] used this approach to find the fiber orientation for a single-layer composite laminate. The problem involved the formulation of the Tsai's failure criterion with respect to the fiber angle, and the solution was expressed in terms of the angle between the fibers and the stress principal directions. This method is very fast and returns all stationary points of the objective function just in one run; however, it works only for single-variable, unconstrained problems with a closed form expression for the objective function. Obstacles may emerge in the formulation of a closed form failure criterion for a general multi-layer laminate, which is the main challenge preventing this method from being extensively used in composite lay-up design.

2.2. Steepest descent (SD)

Steepest descent (SD) is a minimization technique that performs, at each step, a line-search in the opposite direction of the gradient of the objective function. To simplify the problem, SD often resorts to techniques that find an approximation of the ideal step size, which is the one that makes the gradients of the two consecutive successive iterations mutually orthogonal. For laminate stacking sequence design of composite laminates, SD was used in the past either alone [8] or in combination with other optimization techniques [9]. At the onset, it can achieve large variations in the objective function, but as the minimum of the objective function is reached, the convergence rate may become very slow. Besides this limitation, the necessity of working with continuous variables and the drawback of entrapping in local optima are other disadvantages that limit its use for composite lay-up design.

2.3. Conjugate gradient (CG)

Conjugate gradient (CG) methods improve the convergence rate of the steepest descent by choosing conjugate descent directions that are depending on both the gradient of the objective at the current step and the descent directions in the previous iteration. There are different algorithms to perform a conjugate gradient search; among them, Powell's Conjugate Gradient is used by Hirano [10] for buckling load maximization of laminated composite plate under axial compression. The advantage of this method is that no gradient information is required; however, unimodality of the objective function is necessary. For multimodal functions, it is possible to compensate the effect of multi-modality by performing several trials from different starting points [10]. For simple problems where the objective function can be expressed in closed-form, alternative CG methods, such as Powell's Conjugate Directions, may also be used [11]; however, when only an approximation of the gradient is available, techniques such as Fletcher–Reeves or Polak–Ribiere may perform better [12].

2.4. Quasi-Newton method

Due to the requirement of the second-order gradient information, Newton (or Newton–Raphson) methods are rarely used for composite laminate design; however, in their place, the Quasi-Newton (QN) methods are often used, for they allow determining the Hessian without using second-order derivatives.

The QN method by Davidon, Fletcher, and Powell (DFP) [13] is one of the most popular QN techniques used for composite lay-up design. Waddoups et al. [14] and Kicher and Chao [15] used the original form of DFP proposed by Fletcher and Powell [16] to optimize a composite cylindrical shell. A quadratic interpolation of the objective function, which included strength and buckling failure, was used in the one-dimensional minimization process. Kim and Lee [17] also used DFP for optimization of a curved actuator with piezoelectric fibers.

Quasi-Newton methods generally have a higher convergence rate than CG methods, although their performance is problem-dependent and may change from one case to another.

2.5. Method of feasible directions

Method of feasible directions (MFD) is created to solve optimization problems with inequality constraints. Starting from a feasible initial point, MFD tries to find a move to a better point without violating any of the constraints. Since a composite lay-up design problem usually includes several inequality constraints, MFD has been a good candidate for solving these problems [18]. However, like other gradient-based methods, MFD is not always able to find the global optimum.

More recently, with the presence and popularity of finite element methods, MFD has been adapted to be used in combination with finite element analyses [19,20]. In this case, the derivatives are calculated by approximation or direct sensitivity analysis.

MFD may generate non-feasible solutions in the presence of highly non-linear constraints. In order to avoid this problem, Spallino et al. [21] introduced a drive-away factor into the feasibility requirement, by which the search direction was deviated from the constraint boundaries towards the feasible region. Another modification was proposed by Topal and Uzman [22], who took into account not only the gradients of the objective function and the retained active and/or violated constraints, but also the search direction in the former iteration. This modified method, which aims to increase the convergence rate, was used by author researchers to optimize the buckling loads and the fundamental frequencies of a composite part.

2.6. Approximation schemes

An approximation scheme replaces the primary optimization problem with a sequence of explicit approximate sub-problems, each expressed by a first or second-order Taylor series expansion of the corresponding structural function [23]. The benefit is that the sub-problems are solved faster than the original one, while finding the solution of a set of approximate sub-problems should ideally yield to the same solution of the original problem.

Linear approximation is the simplest approximation and was used by Schmit and Farshi [24] to minimize the weight of a symmetric fibre-composite laminate. Here the non-linear programming problem could be transformed into a sequence of linear programs because the stiffness matrix was a monotonous function of the design variables (i.e. thickness of the layers with pre-assigned fiber orientations).

For non-monotonous functions, a mixed approximation was used by Fleury and Braibant [25]. This method, called Conlin's method, consisted of a combination of linear approximation when the corresponding first derivatives were positive and inverse approximation when the first derivatives were negative. A more robust form of Conlin's approximation was developed by Svanberg [26,27] called Method of Moving Asymptotes (MMA), in which the approximate function was obtained by a linearization of the corresponding function with respect to variables of the type $1/(x_i - L_i)$ or $1/(U_i - x_i)$, depending on the sign of the derivatives of the function. In this transformation, x_i is a design variable, and U_i and L_i represent the corresponding upper and lower limits for this variable and are called "moving asymptotes" because they normally changed between iterations. This method was used by Bendsoe et al. [28] to maximize the buckling load factor of composite laminates. In order to achieve a better approximation for a general non-linear function, Svanberg [29] included an additional non-monotonic parameter into MMA formulation, which was updated at each step. The modified method, called Globally Convergence MMA (GCMMA), was recommended to be used when the fiber orientations are considered as design variables.

Bletzinger [30] and Svanberg [29] used the second-order derivative to achieve a MMA approximation that better matches the curvature of the original function. Using a combination of method of diagonal quadratic approximation (DQA) and a MMA by Zhang et al. [31] is another attempt to exploit the benefits of a second-order approximation. The use of second-order derivatives has the advantage of improving the reliability and the efficiency of the optimization process, but it has the drawback of being computationally expensive, especially for large scale problems. To avoid calculating expensive second-order derivatives, Bruyneel and Fleury [32] used the gradient information of the two successive iterations. The modified method called Generalized MMA (GMMA) was employed for the optimization of composite structures where both ply thicknesses and fiber orientations were considered as design variables.

Harte et al. [33] compared the performance of Conlin's method, GCMMA and MDQA in optimizing the layer thicknesses and the winding angle of a glass-reinforced epoxy pipeline. When the variables were limited to only the thicknesses, Conlin's method and MDQA were found to perform equally. Conlin's method required more iterations for convergence, but MDQA was computationally more expensive. When the winding angle was added as a design variable, the only algorithm capable of solving the problem was the GCMMA, although the solutions required a careful examination since they might not be optimal solutions.

Sequential quadratic programming (SQP) methods can also be included in this section. SQP methods handle non-linear problems by constructing and solving a local quadratic program, which consists of a quadratic model of the objective and a linear model of the constraints. SQP has been used for composite laminate design in several applications [34–36].

In comparison to other sequential programming techniques, Wang and Karihaloo [34] reported that SQP produced the most precise results with fewer gradient calculations, but it requires a larger number of functional evaluations compared to SLP (sequential linear programming) and SCP (sequential convex programming). The comparative study was performed on minimizing the stress intensity at a crack tip, and the results cannot be generalized because the accuracy of the approximation depends on the shape of the objective function.

The structural response function of a composite design problem is usually highly non-linear with several local optima, thus an approximation technique can be inaccurate, particularly when the fiber angles are used as design variables. If the accuracy of the approximation is poor, inappropriate results will be obtained. A higher reliability can be achieved by adopting higher order polynomials, which unfortunately are mathematically cumbersome to handle.

3. Direct search methods

While the analytical methods are known for a fast convergence rate, direct search methods have the advantage of requiring no gradient information of the objective functions and the constraints. This feature is a significant advantage because in composite laminate design derivative calculations or their approximations are often costly or impossible to obtain. Direct search methods systematically approach the optimum solution only by using function values from the preceding steps. As a result, several of these techniques have been revealed to be a significant tool for composite lay-up design, as described in the following sections.

3.1. Partitioning methods

A partitioning method, such as Dichotomous search, interval-halving, Fibonacci and Golden section search, is a line search strategy that can handle a single-variable optimization problem. Since composite lay-up design generally copes with several design variables, resorting only to a partitioning method to optimize a composite design is rare. A sporadic example of a single variable problem cited here is the work by Walker et al. [37], who merely exploit the golden section method to determine the fiber angle that best maximizes buckling resistance of a laminated cylindrical shell. Although these methods are rarely used alone, they are effectively used for a line search inside other optimization methods, such as conjugate gradient method and method of feasible directions [12].

3.2. Enumeration search

One of the first attempts in optimum design of laminated composite materials is Enumeration Search, consisted of trying all possible combinations of design variables and simply selecting the best combination. Although cumbersome, this technique was used to find the lightest composite laminate during the 1970s [38,39].

Within this group, we include also selection chart techniques, which aim at visualizing the impact of the variables throughout the whole design space. For instance, Park [40] plotted the change of the optimum fiber angle for a particular class of laminates (e.g. $[-\theta/-\theta]_s$, $[-\theta/90/-\theta]_s$) under all possible normalized loading conditions. Weaver [41] also created selection charts, in which each class of laminates was displayed within an elliptical contour. Useful at the early stage of the design, these charts can be used to identify a small subset of potential laminates which might be investigated in more details. Although selection charts can map the whole design space and give insight on the variables that most impact the performance, their use is limited to design problems governed by a small number of variables and to simple loading conditions.

3.3. Simplex method

The simplex method uses the concept of a simplex, which is a set of $(n+1)$ points in an n-dimensional design space. A simplex, for example, might be a line segment in one dimension, or a triangle in two dimensions. This algorithm starts with an initial simplex, which is generally improved by moving its vertices toward better positions. As described by Nelder and Mead (NM) [42], the improvement is achieved by reflecting the least favourable point of the current simplex with respect to the other points, expand the move if it is favourable, or contract it if it is not favourable. The simplex is scaled down (shrunk) towards the most favourable point if no improvement is achieved. This process is terminated when the simplex becomes smaller than a user-defined size.

NM method was employed by Tsau et al. [43] for optimal stacking sequence design of a laminated composite loaded with tensile forces, while the evaluation of stresses was performed by a finite element method. It has been reported by Tsau and Liu [44] that the NM method is faster and more accurate than a Quasi-Newton method when it is used for lay-up selection of laminates with small number of layers (i.e. less than 4). The error in the simplex method was reported to increase with the number of design variables, as opposed to Quasi-Newton methods that have an almost constant error. Regardless of the number of design variables, NM was found to be faster than a Quasi-Newton method in computational time.

NM method is found to be practical for problems with a small number of design variables (e.g. less than 10), but the convergence rate decreases exponentially with an increase of the number of variables [45]. Other limitations of this method include: being a local and unconstrained optimization method; dependency of the final solution on the initial simplex; and dependency of the convergence rate on the coefficients of expansion, contraction, and shrink. Despite these drawbacks, NM has been widely and efficiently used in several composite laminate problems either separately or in combination with a global method, as discussed later in another section.

3.4. Random and greedy search

A random search evaluates a number of randomly selected points in the design space of a given optimization problem and simply selects the best sampled point, while a track of previously sampled points may be kept by the program to avoid recycling. In contrast to the arbitrary move used in random search, a greedy search evaluates a set of points around the current solution and moves one step in the direction of the best point, the last move is retained until there is no more improvement achieved, then the whole procedure is repeated from the new point.

Foye [46] was the first who used a random search to find the optimum ply orientation angles of a laminated composite plate. Graesser et al. [47] also used a random search, called improving hit and run (IHR), to find a laminate with minimum number of plies that safely sustain a given loading condition. This technique started with either a given or an arbitrary point and sought a new point by randomly changing one or all design variables. Once an improving direction was found, a one-dimensional line search was performed, and the search was continued from the new point. This concept is similar to the one in the Monte Carlo method used by Fang and Springer [3], in which the only difference was retaining the last successful change instead of performing a line search. A more systematic form of this technique is a greedy search method used by Sargent et al. [48].

The advantage of a random search method is its ease of programming and the freedom of selecting a priori the number of iterations. However, the chance of success is strongly dependent on the smoothness of the objective function and on the relative proportions of satisfactory and deficient solutions. In addition, the computational cost to obtain the global optimum exponentially increases with increase of the number of discrete variables [49]. A greedy search is short-sighted and may never reach a better solution if it has to go through a worse solution. Therefore, the success of a greedy search requires a relatively smooth design space and the proper selection of the step size. To overcome partially the limitation of being trapped in local optima, a GRASP (greedy randomized adaptive search procedure), was proposed; however, it has not been used for lay-up design yet.

3.5. Simulated annealing (SA)

Simulated annealing (SA), which mimics the annealing process in metallurgy, globalizes the greedy search by permitting unfavourable solutions to be accepted with a probability related to a parameter called "temperature". The temperature is initially assigned a higher value, which corresponds to more probability of accepting a bad move and is gradually reduced by a user-defined cooling schedule. Retaining the best solution is recommended in order to preserve the good solution [50]. This method is the most popular method after genetic algorithms (GA) for stacking sequence optimization of laminated composite materials [48,51–53].

Generation of a sequence of points that converges to a non-optimal solution is one of the problems in SA. To overcome this shortcoming, modifications of SA have been proposed such as increasing the probability of sampling points far from the current point by Romeijn et al. [54] or using a set of points at a time instead of only one by Erdal and Sonmez [50].

To increase the convergence rate, Genovese et al. [55] proposed a two-level SA including a "global annealing" where all design variables were perturbed simultaneously and a "local annealing" where only one design variable was perturbed at a time. The local annealing was performed after each iteration of the global annealing in order to locally improve the trial point. Its convergence speed was reported higher than a one-level SA and comparable to a gradient-based optimization method implementing a SLP method.

In order to prevent re-sampling of the solutions, Rao and Arvind [56] embedded a Tabu search in SA obtaining a method called Tabu embedded simulated annealing (TSA). The stacking sequence optimization of laminated composites was solved by TSA, while the constraints were handled using a correction strategy. TSA was faster than classical SAs with the penalty of requiring more memory and computation time per iteration.

Simulated annealing is a good choice for the general case of optimal lay-up selection; however, it cannot be programmed to take advantage of the particular properties of a given problem; GA is more flexible in this respect, although it is often computationally more time consuming [48,52,57]. It is not easy to generalize the conclusion because there are some other researches, such as the one by Rao and Shyju [58], claiming that SA outperformed GA both in computational performance and in finding the global optimum for other combinatorial problems.

3.6. Genetic algorithm (GA)

A genetic algorithm (GA) is an evolutionary optimization technique using Darwin's principal of "survival of the fittest" to improve a population of solutions. If the population size is suitably large, GA is not at the risk of being stuck in a local optimum. However, finding a global solution is not necessarily guaranteed to be successful unless an infinite number of iterations are performed. Despite the high computational cost, GA has been the most popular method for optimizing the stacking sequence of a laminated composite [5]. Its simple coding, which escapes gradient calculations, and its flexibility of being applied to a large variety of problems with different types of variables and objective functions make GA particularly useful for problems with multimodal functions, discrete variables, and functions with costly derivatives.

Callahan and Weeks [59], Nagendra et al. [60], Le Riche and Haftka [61], Ball et al. [62] are among the first who adopted and used GA for stacking sequence design of laminated composite materials. GA has been used for several objective functions, such as strength [61,63], buckling loads [61,64–70], dimensional stability [71], strain energy absorption [72], weight (either as a constraint or as an objective to be minimized) [73,52,74–77], bending/twisting coupling [67], stiffness [72,78], fundamental frequencies [1,70,74,79], deflection [76] or finding the target lamination parameters [80].

GA has been applied to the design of a variety of composite structures ranging from simple rectangular plates to complex geometries such as sandwich plates [81], stiffened plates [82,83], bolted composite lap joints [84], laminated cylindrical panels [170]. GA can be often combined with finite element packages that analyze the stress and strain response of the composite structure [85,76,86].

One of the main problems associated with GAs is the high computational intensity and the premature convergence, which may happen if the initial population is not appropriately selected. To increase convergence rate, reduce the risk of premature convergence, and decrease the function evaluation time, several modifications have been proposed. Some of these attempts are summarized here:

- use of parallel computing [87–89];
- multi-level optimization (coarse level and fine level coding) [87];
- introduction of problem-dependent operators [90], such as layer addition or deletion, permutation, interlaminate swap [83], generalized elitist [67], laminate thickness/material/fiber angle mutation [86];
- recovery of previously evaluated solutions [65,75,91];
- use of approximation methods for function evaluation [92,75,69], or training an artificial neural network (ANN) [70,79];
- proposing a consanguineous initial population [66], or a hierarchical structure with aging [93].

Sometimes a combination of these methods are used; for instance, Park et al. [91] used a memory approach combined with the permutation operator with local learning/random shuffling to reduce the number of function evaluation and to improve the convergence rate. They also categorized the design criteria into two groups of layer combination dependent criteria and layer sequence dependent criteria, which recycled some of previously evaluated candidates. However, almost exclusively all these methods are reported to provide an improvement in the GA's numerical efficiency, a comprehensive comparison among them is impossible because GA is a probabilistic method and its performance is strongly dependent on the problem.

GA is originally developed for unconstrained optimization, whereas composite design problems are usually constrained by limitations in material's strength, weight, cost or other criteria. Among several methods used to incorporate constraints, using a penalty function strategy is the most popular one. A penalty function is introduced to convert a constrained problem into an unconstrained problem by assigning a penalty term to the infeasible solutions. The penalty assigned to each solution depends on number and intensity of violated constraints. Another method to handle the constraints is the repair strategy [80], which transforms infeasible solutions into a feasible solution located in a close proximity.

Sargent et al. [48] compared GA with some greedy algorithms (i.e. random search, greedy search, and simulated annealing) and observed that GA produced better solutions than the greedy searches, which in some instances were unable to find a solution. Although more demanding in terms of computation time, GA revealed to be a more robust algorithm. Sivakumar et al. [94] compared DFP and GA, both applied to minimize the weight of a laminated composite constrained by its fundamental frequencies. It was reported that DFP converged in smaller number of iterations when the number of constraints was small; however, finding a feasible point was a difficult task when number of constraints increased. Also considering that the DFP could not handle discrete variables, they concluded "GA seems to be the best tool to optimize composite laminates".

Although GA has been widely used for stacking sequence optimization, one major shortcoming is its low convergence rate. GA is a population-based evolutionary algorithm, and might require several generations before converging to a solution [58]. Each generation consists a large number of function evaluations, thus it can be computationally time consuming and expensive.

3.7. Other heuristic optimization methods

Although GA and SA are the most popular heuristic methods used to optimize the stacking sequence of composite laminates, other heuristic methods have been also used. This section summarizes the main ones.

3.7.1. Tabu search

Tabu search (TS) is a local search method that starts from an initial point and progresses by changing the design variables, one at a time. The entire or a part of the neighbourhood is evaluated before accepting the best solution as a starting point for the next move. TS keeps, in a short term memory, the potential solutions that have already been visited and marks them as "Tabu". This strategy prevents solution re-sampling. TS was implemented by Pai et al. [95] for discrete optimization of the stacking sequence of a composite laminate subjected to buckling and strength requirements as well as for matrix cracking. The results were compared with a GA, which showed a comparable solution, but the computational time was case dependent.

3.7.2. Scatter search

Scattered search (SS) is a strategy that generates a reference set from a population of randomly selected solutions. From this reference set, a subset of solutions is selected and enhanced by using an improvement procedure (e.g. a greedy search). The improved solutions are then used to update the reference set, and the process is continued with a new subset. Rao and Arvid [96] used a scatter search for lay-up sequence optimization including thermal buckling, weight, cost, fundamental frequency and buckling considerations. Lay-up results and computational efficiency were similar to those obtained with GA.

3.7.3. Particle swarm

Like GA and SS, particle swarm optimization (PSO) is also a population-based, stochastic optimization method inspired by the flocking behaviour of birds. In this method, each solution in the search space is called a "particle" and resembles a bird among others, which adjusts its position in the search space according to its own flying experience (best solution in its individual history) and the flying experience of the other particles (the best solution among all particles). As such, PSO has a good potential to benefit from parallel computing.

PSO was used by Suresh et al. [97] for the optimal design of a composite box-beam of a helicopter rotor blade. It was reported that the solutions provided by PSO were closer to the optimum values than those given by the GA. The computational time, on the other hand, was comparable. Kathiravan et al. [98] compared PSO to a gradient-based method for the maximization of the failure strength of a thin walled composite box-beam, whose design variables were the ply orientation angles. The PSO was found to give results superior or equivalent to the gradient-based method. Furthermore, it did not need to start from different initial points.

Chen et al. [99] introduced two improvements to PSO. First, a random coefficient was added that increased the individual influence and the swarm variety and improved the search ability. Second, an interference method was proposed to overcome the lack of local exploration at later stages of the search, particularly when the objective function has multiple local optima. This modification was applied only if the results remained unchanged during a certain number of consecutive iterations. The improved method, which was tested on lay-up stacking sequence design, was reported to converge faster and to be more stable than the original PSO.

3.7.4. Ant colony

Ant colony optimization (ACO) is another heuristic search method, which is inspired by the behaviour of the ants and their ability in finding the shortest paths between their nest and the food source. Aymerich and Serra [100] illustrated the application of this technique to the lay-up design of laminated panels for maximizing the buckling load. The average performance of the ACO, as measured by practical reliability and normalized price, was evaluated "comparable or even superior" to GA or TS techniques.

4. Specialized algorithms

In this category, fall methods developed explicitly for optimizing laminated composite materials. These strategies exploit a number of properties of the composite laminates to simplify the optimization process. Often developed for a particular application, these methods generally simplify the problem by restricting the design space in terms of allowable lay-up, loading condition, and/or the objective function. Since these methods are tailored to a specific design problem, they lose robustness when applied to a general optimization problem. However, if used for the particular case they are designed for they can be much faster than other techniques.

4.1. Design with lamination parameters

Using lamination parameters [101,102], which are integrated trigonometric functions through the thickness of a laminate, instead of lay-up variables has the big advantage of reducing the number of parameters required to express a laminate properties to maximum of 12, regardless of number of layers [2]. Avoiding troublesome optimization over periodic functions of the rotation angles and discrete number of plies is the other advantage of using lamination parameters. The convexity of the set of lamination parameters together with the linear dependence of the stiffness on these parameters implies further simplification with respect to the basic mathematical structure of the problem [103].

Beside the promising advantage of using lamination parameters, the challenge in working with these parameters is that they are not independent and cannot be arbitrarily prescribed. The admissible range of lamination parameters are given by solving the geometric relations. Several authors, such as Fukunaga and Vanderplaats [104] and Grenestedt and Gudmundson [105], have suggested necessary conditions for different combinations of lamination parameters, but the complete set of sufficient conditions for all twelve parameters is still unknown [103].

The lamination parameters strategy requires solving the inverse problem to obtain the corresponding number of plies, thicknesses and fiber orientations, which are suitable for manufacturing. Solving the inverse problem is not easy and the solution is not unique. Miki [106] proposed a method to visualize the admissible range of lamination parameters and their corresponding lay-up parameters. The method is fast and handy, although it can be applied only to the laminates with prescribed in-plane stiffness properties and balanced-angle-ply laminates of type $[\pm\theta_1/\pm\theta_2/\ldots/\pm\theta_i]_S$. Just as for the in-plane lamination diagram, the flexural lamination diagrams were also developed [107]. Fukunaga and Chou [108] used a similar graphical technique for laminated cylindrical pressure vessels. Lipton [109] has developed an analytical method to find the laminate configuration of a three-ply laminate under in-plane loading conditions. Autio [110], Kameyama and Fukunaga [111], Enrique et al. [112], and some other researchers used GA to solve the inverse problem.

In general, optimization methods based on the lamination parameters are restricted to global structural responses, such as stiffness, and do not include local strength constraints at the ply level. Optimal design studies are limited to particular structural responses and to specific laminate configurations. Inverse problem must be solved in order to get the corresponding number of plies, thicknesses and fiber orientations, which is the most challenging part of working with lamination parameters.

4.2. Layerwise optimization

A layerwise optimization method optimizes the overall performance of a composite laminate by sequentially optimizing one or some of the layers within a laminate. This method works with one layer or a subset of layers in the laminate and requires first the selection of the best initial laminate and then the addition of the layer that best improves the laminate performance, which is usually achieved by an enumeration search [113,114].

Lansing et al. [9] determined the initial laminate by assuming the layers at 0, 90, and ±45° carrying all the longitudinal, transverse, and shear stresses, respectively. Massard [115] started with a one-layer laminate and found the best fiber orientation for the single-ply laminate. Todoroki et al. [116] proposed two other approaches to find the initial laminate. They first used quasi-isotropic plies to estimate the initial number of layers; the second determined the lower bond for the number of layers by using a super-ply. A super-ply is a virtual layer that had the best material properties in all directions.

An important issue in layerwise optimization strategy is the impact of the newly added layer on the stress distribution of the laminate. The addition of a new layer changes the stress field in the laminate, because there is a change in the position of the layers in the laminate and in the portion of load each layer can sustain. Therefore, layers inserted in previous steps are no longer optimal. Narita [117] and Narita et al. [118] tried to solve this problem by starting with a laminate with hypothetical layers with no rigidity. From the outermost layer, all layers were sequentially replaced by an orthotropic layer and the optimum fibre orientation angle was determined by enumeration. The first solution was then used as an initial approximation for the next cycle. Farshi and Rabiei [119] proposed a method for minimum thickness design consisting of two steps. The first aimed at introducing new layers to the laminate. The second examined the probability of replacing the higher quality layers with weaker materials. Ghiasi et al. [120] used layer separation technique to keep the location of different layers unchanged when a layer is added. A new layer was added to the laminate by dividing one of the current layers into two layers. Then a scaling procedure was used to change the thicknesses.

These methods are very fast compared to other optimization techniques, because they are working only with a small portion of all the design variables in each step; however, they may not reach a local optimum. This problem can be partially solved by revising the fiber orientations and thicknesses every time a new layer is introduced to the laminate. Nevertheless, the additional computational time and cost induced by this approach might be excessive and of limited value since it will only assure achieving a local optimum.

4.3. Problem partitioning

Composite lay-up design is governed by variables of different nature (e.g. thickness and fiber angle), which increase the complexity of the design problem. To simplify the design problem, one possible approach is to split the problem into some dependent sub-problems, each described with a small number of variables of similar nature. This section gives an overview of these techniques.

Two-part methods are the most common form of problem partitioning, in which the first part finds the fiber direction that provides optimum property such as stiffness, displacement, natural frequency, etc., and the second part, optimizes ply thicknesses for minimum weight [121,122]. Le Riche and Gaudin [70,71] added one more sub-problem to this technique, which includes minimization of the total number of layers in the laminate. Farshi et al. [123] also used three sub-problems, but embedded with a layerwise optimization. In this method, a new layer was added to the stack after optimization of fiber angles and then thicknesses, which was repeated until the thickness of the recently added layer approached zero.

Partitioning the problem may also be used to improve the GA's performance, because when both real and discrete variables are represented using a single string of GA the cost of the search is increased. After partitioning, sub-problems with different type of variables can be represented with different strings or can be solved by different optimization methods [124,125]. In general problem partition can significantly increase the convergence rate; however, since the sub-problems are not independent, the final solution in certain instance might be far from the global optimum.

4.4. Discrete material optimization (DMO)

Lund and Stegmann [126] used a method called discrete material optimization (DMO) for optimum design of composite laminates. In this method, which can be equally applied to constant and variable stiffness design, the material stiffness is computed as a weighted sum of some candidate materials (fiber orientations). The discrete problem of choosing the best material (with the right orientation) is converted to a continuous formulation where the design variables are the weight functions on each candidate material. For each ply, the goal is to drive the influence of all but one of these weight functions to zero. A disadvantage of this method is that it replaces each design variable (fiber angle) with several weight factors, each corresponding to one permissible fiber angle.

4.5. Fractal branch-and-bound method

Fractal branch-and-bound (FBB) method, proposed by Todoroki and Terada [127,128], and Hirano and Todoroki [129,130] is based on branch-and-bound method, whose computational cost is reduced using a response surface to approximate the objective function in terms of lamination parameters. This technique performs an evaluation of the objective function to prune inefficient branches (laminates) by utilizing the fractal patterns of feasible region of lamination parameters in symmetrical laminates. Later on, FBB was expanded to unsymmetrical laminates by Matsuzaki and Todoroki [131]. It has also been adopted for complicated structures with multiple laminates such as hat stiffened panels [132].

4.6. Knowledge-based methods

Experimental or knowledge-based methods (KBM) aims at bringing the expert's knowledge into the computer design process by performing a screening of the concepts ill-suited to the specific requirements and focusing quickly on the most viable and attractive alternatives. To produce a short list of good layups, these methods may use different tools, such as discrimination charts and tables eliminating unsuitable arrangements [133] or a list of rules and recommendations for good lay-ups [134,135]. A KBM can be faster than a numerical method because it reduces the number of candidates at the onset, but it may not explore the entire range of potential layups [134]. In addition, the process of developing such knowledge-based systems is problem-dependent and needs a great deal of experience.

5. Hybrid methods

A hybrid method combines two or more optimization methods to benefit from the advantages of all of them in order to obtain a better convergence rate, to achieve a global optimum, to have a better accuracy, to make the optimization method more robust, or for other reasons.

In order to handle both continuous and discrete design variables in a general multimodal non-linear problem, Seeley and Chattopadhyay et al. [136] developed a combined simulated annealing and sequential programming optimization techniques (more specifically BFGS). The method was applied to the design of a scaled airplane wing model, represented by a flat composite plate, with piezoelectric actuation to improve the aeroelastic response.

Incorporating a local optimization technique into GA [65,81] is the most popular hybrid approach, in which a local search algorithm is applied to some newly generated individuals to drive them to a local optimum. These locally optimal solutions replace the current individuals in the population to prepare the next gen-

Table 1
Properties of different optimization methods used for composite lay-up design.

Method name	Method	Global	Continuous	Discrete	Constrained	Derivative	Deterministic	Relative convergence rate
Vanishing the function gradient	D0		X			2	X	
Steepest descent	SD		X			1	X	
Conjugate gradient								
Linear conjugate gradient	LCG		X			1	X	SD+*/in n steps
Powell's method	CG-P		X				X	SD+/in n steps
Fletcher–Reeves	CG-FR		X			1	X	Aprx. in n steps
Polak–Ribiere	CG-PR		X			1	X	CG-FR+
Quasi-Newton								
Broyden–Fletcher–Goldfarb–Shanno	BFGS		X			1	X	Super linear
Davidon–Fletcher–Powell	DFP		X			1	X	Super linear
Method of feasible directions	MFD				X	1	X	
Modified feasible direction (Topal)	MFD		X		X	1	X	MFD+
Conlin's approximation method	Colin		X		X	1	X	
Method of moving asymptotes	MMA		X		X	1	X	
Globally convergent MMA	GCMMA		X		X	1	X	
Globally convergent MMA-2	GCMMA2		X		X	2	X	GCMMA+
Generalized MMA	GMMA		X		X	1	X	GCMMA+
Method of diagonal quadratic approximation	MDQA		X		X	2	X	GCMMA+
Enumeration search	ES	X		X	X		X	
Partitioning methods							X	
Golden section	GS		X		X		X	
Simplex method								
Nelder–Mead simplex	NM		X				X	
Random search	RS	X		X	X			~ES**
Monte Carlo	MC	X		X	X			RS+
Improving hit and run	IHR	X	X	X	X			~MC
Greedy search	GRS		X	X	X		X	RS+
Greedy randomized adaptive search procedure	GRASP	X	X	X	X			~GRS
Simulated annealing	SA	X	X	X	X			~GRS
Improved SA	ISA	X	X		X	1		~SLP/SA+
Genetic algorithm	GA	X		X	X			
GA + local improvements	GA+L	X		X	X			GA+
GA + special operators	GA+O	X		X	X			GA+
Scatter search	SS	X	X	X	X			~GA
Tabu search	TS		X	X	X			
Particle swarm optimization	PSO	X	X	X				
Ant colony optimization	ACO	X	X	X				
Knowledge-based methods	KBM		X	X	X		X	

* Having a better convergence rate.
** Having almost a similar convergence rate.

eration. This combination utilizes the advantage of GA in finding the global optimum and the advantage of a local optimization method in quickly arriving to an optimum solution. This hybrid optimization technique needs less number of function evaluation compared to pure GA.

Another common form of hybridization is to restart a local optimization technique, several times. A good example is the globalized bounded Nelder–Mead (GBNM) by Luersen and Le Riche [137]. This method was particularly adapted to tackle multimodal, discontinuous optimization problems, for which it is uncertain that a global optimization can be afforded. The algorithm consists of several restarts of a local search, performed by the Nelder–Mead simplex method, each initiated from a simplex likely far from previously sampled points. The proximity to previously sampled point was controlled by a multi-dimensional probability function, which has been later simplified to a one-dimensional function by Ghiasi et al. [138] in order to reduce the computational time and improve the convergence rate. The local-global search has been shown to converge faster than an evolutionary method, such as GA when the number of iterations is small; however the two methods perform similarly, when the number of iterations increases.

Oh and Lee [139] used the same technique, but replacing the probability function with a domain elimination technique, which consisted of making a set of eliminated regions, and terminating the local search when the current search is adjacent to an eliminated region.

Rao and Shyju [58] integrated SA and TS into an algorithm called multiple start guided neighbourhood search (MSGNS). The multi-start scheme helps overcome the sensitivity of SA on the cooling schedule, while the Tabu search prevents recycling the previous solutions. For stacking sequence optimization of laminated composites, MSGNS was found to be as effective as GA in obtaining multiple near optimal solutions, and superior in terms of computational performance to algorithms like GA, SA, and SS.

6. Concluding remarks

The most popular methods for optimizing the stacking sequence of laminated composites with uniform stiffness have been reviewed in this paper. The optimization techniques are classified into four categories: gradient-based methods, direct search and heuristic methods, specialized techniques, and hybrid methods. A brief list of all these methods and their properties is shown in Table 1. The last column in this table shows the relative convergence rate of each method compared to others, when such data are available in literature.

Gradient-based methods are found to be generally faster than other techniques and can find a local minimum in a small number of iterations. These methods, however, are limited to problems with continuous design variables and first or second derivatives (either exact or approximate). Since they reach local optimum, the final solution depends on the initial point. If the problem has a small number of design variables and the objective function is smooth, then gradient-based methods are the best choice, especially if several restarts from different initial points can be afforded. These methods do not find large application to composite lay-up design because gradient information is often either unavailable or very expensive to determine.

In contrast, direct search methods are very popular because no derivatives need to be calculated. These methods can be divided into deterministic and stochastic. Stochastic methods are more appropriate for composite lay-up design, because of their capabilities of handling a mixture of continuous and discrete variables, finding the global optimum of a multi-modal objective function, and working with a population of solutions. These methods usually have low rate of convergence, which is also a problem dependent factor. Consequently, comparing stochastic methods is unfeasible because their heuristic nature make them strongly dependent on the problem. Until now, Genetic Algorithm has been the most popular method with simulated annealing ranked the second.

The next group of methods called "specialized methods" includes a variety of techniques developed for the design of a specific type of laminates, loading conditions, and/or objective. These methods are usually fast, but are limited to certain types of problems and can find only a local minimum.

Finally, hybrid methods benefit from the advantages of all their constituent methods. The review of the main hybrid methods in this paper has highlighted their promising future.

Optimization methods for variable stiffness design will be the focus of a second paper, which will review also other issues, such as multi-objective optimization, discretization techniques, design for manufacturing, sensitivity analysis, and design for uncertainty.

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