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Analysis of the structural efficiency of trees

S. C. BURGESS[†]* and D. PASINI[†]

This paper presents an analysis of the structural features of trees in order to understand how high levels of mass-efficiency are achieved and to identify lessons for engineering designers. Structural features are identified from a literature survey and also by observations and measurements of mature trees in the UK. The functions of the structural features are identified using a function-means tree. The mass-efficiency of the structural features is modelled and assessed using first-order stress analysis. A comparison is made between the structural features found in trees and those used in engineering. There are many similarities between the structural features in trees and engineering. However, there are a few aspects that are mostly unique to trees such as the existence of important non-structural functions in structural members and the presence of adaptive growth. These features may indicate how future engineering structures will be designed.

1. Introduction

Nature contains high levels of optimum design, and engineers have often drawn important lessons and ideas from nature (Thompson 1961, French 1988, Vogel 1998). Examples of recent research studies in nature have included deployment systems in plants (Vincent 2000), sustainability studies in nature (Thompson 1999) and reliability strategies in nature (Burgess 2002). This paper presents an analysis of the structural features of trees in order to understand how high levels of mass-efficiency are achieved and to see whether there are lessons for engineering designers.

Trees have very impressive structural statistics. Trees are very tall structures and are in fact the tallest organic structures in nature. The record for height is currently held by the Giant Redwood trees at around 120 m. Trees are also the heaviest organic structures in nature. The heaviest known living tree is the Sequoia tree that weighs up to 2000 tonnes. Trees are also the longest living structures in nature, with some trees such as the Bristlecone pines living up to 5000 years. Trees also survive in a wide range of environments including hot, cold, dry, wet, and windy environments. These statistics indicate that trees must contain efficient and sophisticated structures. The successful application of wood as an engineering material for many centuries also indicates that trees are important structures.

In modern times, trees have been discovered to be 'smart' structures that can adapt to their environment. Trees are able to sense light and grow branches towards light (i.e. away from the shade of other trees or obstacles) in order to maximize their ability to

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† Bristol University, Department of Mechanical Engineering, Queens Building, University Walk, Bristol BS8 1TR, UK. e-mail: s.c.burgess@bris.ac.uk
* To whom correspondence should be addressed.

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produce food by photosynthesis. Trees can also adapt the size and shape of their structures in response to changes in loading conditions. When living wood experiences high stress levels, it responds by growing more cells in that area in order to reduce stress levels (Mattheck 1989). With current interest in smart structures, this gives added motivation for the study of trees.

Trees have been subjected to many studies in recent times. The microstructure of wood has been modelled and compared with engineering materials (Ashby *et al.* 1995). The performance of roots as anchoring systems has been assessed (Coutts 1983a, 1986, Stokes *et al.* 1995). Investigations have been carried out on the ability of trees to adapt to windy conditions (Coutts 1983b, Wood 1995). The ability of trees to adapt the shape of their branches has also been described recently (Pasini and Burgess 2002).

However, no detailed study has been carried out on the mass-efficiency of the structural shapes and forms found in trees. In addition, no detailed comparison has been made between the structural features of trees and those used in engineering. This paper addresses these points. The efficiency of form and shape is modelled using first order stress analysis.

2. Analysis of tree functions

In order to assess the 'fitness for purpose' of the structural features of trees, it is important to clarify the actual purpose of those features. The functions of the structural features of trees were investigated using a function-means tree. A function-means tree starts off with a high-level objective and shows how this high-level objective is fulfilled by progressively lower level objectives and means. Function-means trees can be very useful in clarifying the purpose of the different parts of complex systems (Robotham 2002).

It is not possible to produce a definitive high-level function for a tree because it depends on the location of the tree. For the purposes of this study, a high-level function of a tree was assumed to be 'successfully compete with other trees'. A tree is a very complex biological system and so a function-means tree can be very large and complicated. In order to make the function-means tree manageable, there was a focus on the functions of the structural features of trees.

Using the high-level function of 'successfully compete with other trees', a function-means tree was produced as shown in figure 1. The top sections of the tree show how high-level functions are achieved by low-level functions. The lower sections of the tree show the means by which the functions are achieved. The diagram also shows how high-level means are achieved by low-level means.

The main functions that enable the high-level function of the tree to be met are the production of sufficient food and the production of seeds to enable reproduction. The function that is of interest in this study is that of producing sufficient food. Therefore, only this function is developed further in the function-means tree.

The means by which a tree produces sufficient food is by the process of photosynthesis and this is carried out by chlorophyll in the green leaves. Photosynthesis can be summarized as the conversion of carbon dioxide, water and light energy into carbohydrate and oxygen. The oxygen is released to the atmosphere and the carbohydrate is used by the tree for food. To obtain carbon dioxide does not require the tree to have a particular position or orientation. However, to obtain sufficient light and water

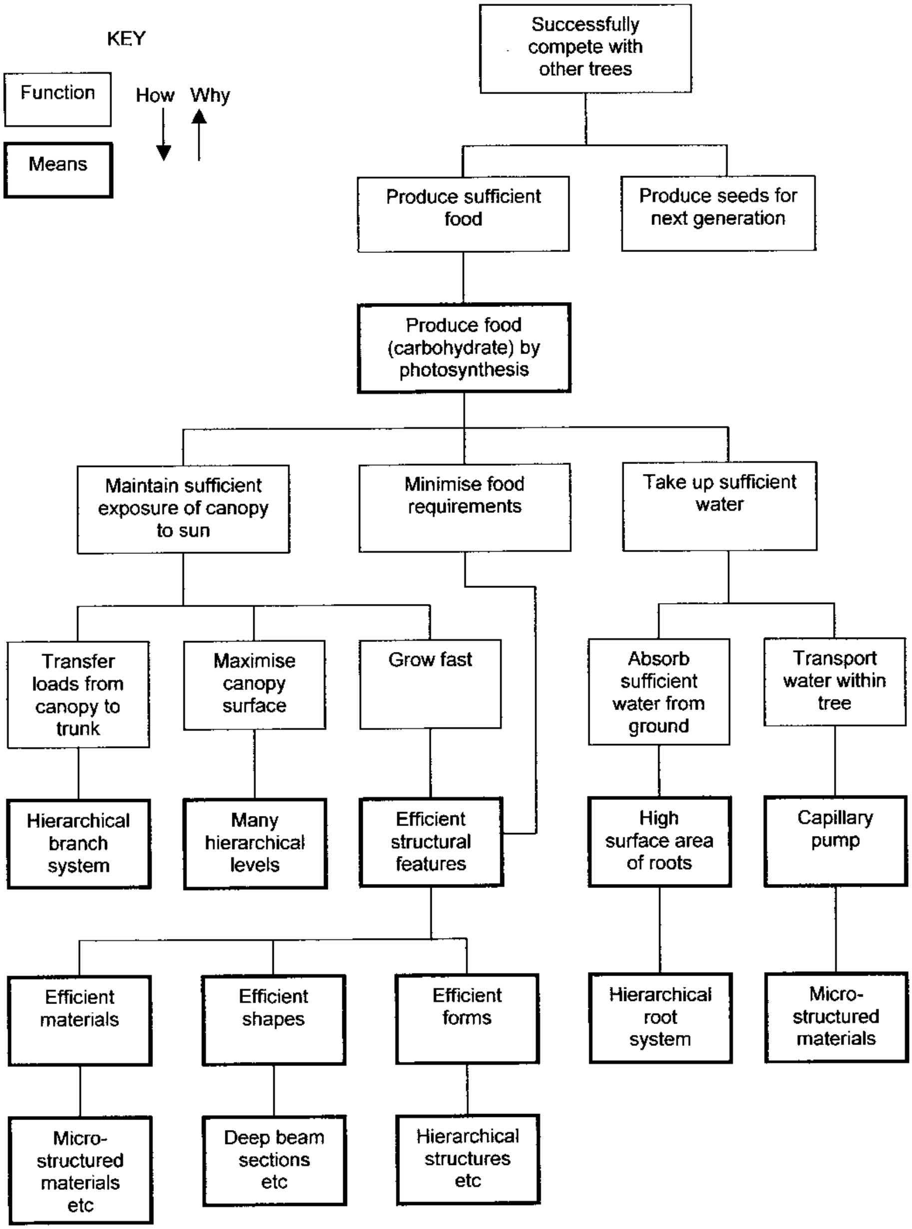

Figure 1. Function-means tree for the structural features of a tree.

does require the tree to have a favourable position and orientation. In particular, the tree must have a minimum amount of leaves exposed to the sun and it must have a minimum amount of roots with access to water.

All of the structural features of trees relate directly or indirectly to giving the tree adequate access to light and water. For example, a key function of a tree is to have a sufficiently large canopy of leaves. Therefore, the tree must have a system of supporting a canopy. The function of any structure is to transfer loads from one position (or set of positions) to another position (or set of positions). The tree has an interesting problem of transferring loads from a non-flat surface of leaves to a central column. In order to transfer loads from a surface to a central column, the tree uses a hierarchical branching system.

Another important function of the tree is fast growth. Fast growth is necessary so that the tree is not put in the shade by other trees. One of the ways that a tree can grow fast is to use a minimum of material to achieve height. In order to minimize the amount of material, the tree requires mass-efficient structural features. The use of mass-efficient structural features can make the difference between survival and extinction for the tree.

One interesting aspect of design that is revealed by the function-means tree is that some structural features are advantageous for more than one structural reason. For example, efficient structural features are good not only for achieving fast growth, but also for minimizing the food requirements of the tree and also for minimizing the loads due to self-weight. A structural hierarchy is not only a good solution for maximizing the surface of the canopy, but it also an efficient means for doing this.

Another interesting property of trees that is clearly revealed by the function tree is that some of the structural features have additional functions that are not structural. For example, the micro-structure of the wood material is mass-efficient from a structural view-point but it also has an important function of pumping water by capillary action. The high surface area of roots gives high structural strength. However, the large root structure is also required to enable the tree to draw sufficient water. Therefore, it must be remembered that many parts of a tree are optimized for multiple functions and not just structural functions.

3. Principal loads and scale effects

It is important to clarify the main sources of loading on a tree in order to understand the detailed functions of the different structural features. There are two main sources of loading on a tree: wind and self-weight. The wind causes bending loads in the trunk and the branches. Self-weight causes mainly compressive stresses and buckling loads in the trunk. In addition, self-weight causes bending loads in all branches that do not have a vertical orientation. Since trees gradually grow into large structures during their lifetime, it is important to consider the effects of scale on tree loading. These aspects are considered in the following sections.

3.1. Bending stresses in the trunk due to wind loading

In general, the wind blows on a tree in a lateral direction, with the wind speed increasing with increasing height. Since a tree is a fixed object, the wind causes an aerodynamic force on the tree. At high wind speeds, the Renolds number is very high so the drag is dominated by form drag and viscous friction is relatively low. An estimation of the aerodynamic force due to the wind, F_{w}, is given by:

$$F_{\mathrm{w}} = 0.5 \rho A_{\mathrm{f}} C_{\mathrm{d}} v^2 \tag{1}$$

where ρ is the density of air, A_{f} is the frontal area of the tree in the direction of the wind, C_{d} is the drag coefficient and v is the velocity of the wind.

The aerodynamic force causes bending moments in the tree that vary from zero at the top of the trunk to a maximum at the bottom of the trunk. The maximum bending stress at the bottom of the trunk is given by:

$$\sigma_{\mathrm{b}} = \frac{M}{Z} \tag{2}$$

where M is the maximum bending moment and Z is the section modulus (which is equal to the second moment of area, I, divided by the distance to the neutral axis, y). For a circular section of diameter d, the section modulus is given by:

$$Z = \frac{\pi d^3}{32} \tag{3}$$

In order to appreciate the importance of wind loading, the following analysis makes an estimate of the maximum bending stress at the root of a trunk of height h and trunk diameter d_{t} for a wind velocity of v. It is assumed that the wind has a centre of pressure that acts two-thirds the height of the trunk from the ground.

For these conditions, and using equations (2) and (3), an estimation of the maximum stress at the root of the trunk is given by:

$$\sigma_{\mathrm{b}} = \frac{F_{\mathrm{w}} 2h}{3} \frac{32}{\pi d_{\mathrm{t}}^3} \tag{4}$$

In order to estimate a bending stress it is necessary to make an assumption about the size of the tree canopy. The following analysis assumes a frontal area of canopy equivalent to a circle of radius $0.75h$. This representative tree geometry is shown in figure 2. Such a canopy size is representative of deciduous trees. Based on this assumption, the maximum bending moment is given by:

$$\sigma_{\mathrm{b}} = \frac{3}{2} \rho C_{\mathrm{d}} v^2 \frac{h^3}{d_{\mathrm{t}}^3} \tag{5}$$

Equation (5) shows that, for a given wind speed, the maximum bending stress is independent of the scale of the tree. Even though the aerodynamic force and moment arm

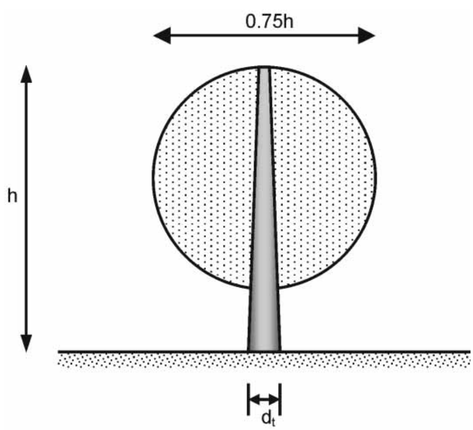

Figure 2. Representative tree geometry.

of that force both increase for larger trees, this effect is exactly counteracted by the increase in the second moment of area of the trunk. In practice, wind speeds increase with height above ground and therefore taller trees generate slightly larger bending stresses than smaller trees if h/d_t remains constant. It should be noted that most trees have residual stresses in their trunk and for such trees this has to be taken into account in order to estimate the net level of bending stress. This aspect will be discussed in section 5.2.

Table 1 presents the maximum stress versus wind speed for a tree that has a trunk diameter at the base of the tree of between $d_t = h/20$ and $d_t = h/40$. The results are obtained from equation (5) assuming that $\rho = 1.2\,\text{kg/m}^3$ and $C_d = 1$. Since the strength of wood is typically in the range of between 25 MPa (balsa wood) and 145 MPa (lignum wood), the bending stresses caused by high winds are very significant. In storms of more that 100 m.p.h. the trunks and branches of trees that are exposed to the wind are very vulnerable to braking, especially if there are any weaknesses present.

It should be noted that the assumed canopy size is only representative of deciduous trees and not coniferous trees. The results in table 1 help to explain why deciduous trees with broad canopies normally have a relatively thick trunk. In contrast, conifer trees are able to have a much slender trunk because of their much smaller canopy.

3.2. *Compressive stresses in trunk due to self-weight*

One of the effects of self-weight is to produce compressive stresses in the trunk. For a trunk that tapers linearly from d_t at the root of the trunk to $d_t/2$ at the top of the trunk and where the combined weight of all the branches weigh one-half as much as the trunk, the weight of the tree is given by:

$$P_w = \frac{75}{512}\pi d_t^2 h \rho g \tag{6}$$

where ρ is the density of the wood and g is the gravitational constant. The compressive stress as the root of the trunk is given by:

$$\sigma_c = \frac{75}{128} h \rho g \tag{7}$$

From equation (7) it can be seen that the compressive stress at the root of the trunk increases linearly as the tree grows in height. For a 10 m high pine tree of density

Wind speed (m.p.h.)	Trunk diameter, d_t		
	$h/20$	$h/30$	$h/40$
40	5	16	37
60	10	35	83
80	18	62	147
100	29	97	230

Table 1. Bending stress (MPa) at the root of the trunk for different wind speeds.

$\rho = 600\,\text{kg/m}^3$, equation (7) gives a compressive stress of 0.04 MPa. For a 30 m pine tree equation (7) gives a stress of 0.11 MPa. From this first-order analysis it is clear that compressive stress due to self-weight is not significant, even for large trees.

3.3. Buckling of trunk

The critical Euler buckling load, P_c, for a straight vertical cantilever of height h that is unrestrained at the top is given by:

$$P_c = \frac{\pi^2 EI}{4h^2} \tag{8}$$

where E is Young's modulus and I is the second moment of area. In practice, a tree trunk is never straight and without defects so the critical load could be significantly less than this critical load.

The load on a tree trunk that can cause buckling is caused by the weight of the tree. Of course, the weight of the tree does not act at the top of the tree. In order to assess the resistance to buckling using the Euler formula, it would be necessary to estimate the point load that gives the equivalent affect of the distributed load.

Table 2 presents the critical Euler load and tree weight for different tree heights using equations (6) and (8). The results in table 2 assume values of $E = 11$ GPa and $\rho = 600\,\text{kg/m}^3$, which are typical for pine. In addition, it is assumed that $d_t = h/30$. Table 2 also presents the ratio of the critical Euler load to tree weight. This ratio gives an indication of the stability of the tree. The results in table 2 show that the ratio of critical load to tree weight becomes worse as the tree grows taller. When account is taken of the affects of eccentricity and defects in the trunk, it is clear that there may be little or no safety factor against buckling for trees over 100 m in height. Indeed, buckling may be one of the factors that places a physical limit on the height of trees.

3.4. Bending stresses due to self-weight

The tree trunk usually experiences no significant bending stresses due to the self-weight of branches because the trunk grows nearly vertically upwards and the weight of branches on one side of the trunk is generally counterbalanced by the weight of branches on the other side of the trunk. Even though individual branches cause local bending stresses on the trunk, the net combined effect of the branches is that there are no significant bending stresses at the root of the trunk.

However, significant bending stresses are experienced in large horizontal branches due to the self-weight of those branches. The maximum bending stress in a cantilever is given by equation (3). For a circular branch of length L and assuming a constant

Height, h (m)	20	40	60	80	100	120
Tree weight, P_w (MN)	0.02	0.20	0.66	1.57	3.07	5.30
Critical Euler load, P_c (MN)	0.66	2.63	5.92	10.5	16.4	23.7
P_c/P_w	33	13	9.0	6.7	5.3	4.5

Table 2. Critical Euler buckling loads and tree weights versus tree height.

taper from a diameter of d_b at the root of the branch to zero at the tip of the branch, the maximum bending moment at the root of the branch is given by:

$$M = \frac{\pi d_b^2 L^2 \rho}{48} \tag{9}$$

The section modulus of a circular section is given by equation (4). Therefore, the maximum bending stress is:

$$\sigma_b = \frac{2L^2 \rho}{3d_b} \tag{10}$$

From equation (10) it can be seen that as the scale of the branch increases, the bending stresses increases because there are two dimension terms on the numerator and only one in the denominator. In contrast, the load on the branch due to aerodynamic forces remains constant as the branch scales in size as shown by equations (1) and (4). Therefore, as the branch grows in size, self-weight becomes more dominant compared with aerodynamic loading. The effect of scale on the importance of self-weight will be described in detail in section 6.1.

4. Structural features in the overall form of the tree

4.1. Structural hierarchy

One of the most important structural features of trees is their structural hierarchy. There is a structural hierarchy of branches from the trunk up to the leaves and there is a structural hierarchy in the leaves themselves. There is also a structural hierarchy of roots from the base of the trunk down into the ground. A typical structural hierarchy from the trunk to the canopy of leaves can be described as follows:

- Level 1: Trunk.
- Level 2: Main branches.
- Level 3: Secondary branches.
- Level 4: Tertiary branches.
- Level 5: Leaf stem.
- Level 6: Leaf main ribs.
- Level 7: Leaf secondary ribs.
- Level 8: Leaf webs.

The advantages of a hierarchy in a structure can be summarized as follows:

i. *Forms a system of connections between a point and a large surface area.* A structural hierarchy is inherently suited to connecting a point source with a surface and therefore a structural hierarchy is inherently suited to connecting a trunk with a canopy.

ii. *Creates a relatively direct load path.* When there is a structural hierarchy, loads from the canopy can be transported in the most direct path towards the trunk. If there is little hierarchy (e.g. there are no secondary and tertiary branches), it is inevitable that loads will take a more tortuous path and this is inherently less efficient due to increased bending moments.

iii. *Produces less critical buckling loads.* A structural hierarchy results in less severe buckling loads because the length of individual members is much less and the load in individual members is much less. The reduced level of critical buckling loads means that less material is required to withstand buckling.

iv. *Inherently suited to gradual growth.* A structural hierarchy is inherently compatible with the gradual growth of the tree structure.

The branching system of trees shows that a hierarchical structure with many layers leads to high structural efficiency. It is interesting to note that intricate branching systems are seen in many places in nature including, river tributaries, blood circulation systems and lightning. Bejan (2000) has shown how intricate branching systems are usually the most efficient means for connecting a point source with a surface.

Hierarchies are often seen in engineering structures such as bridges and buildings. It is interesting to note that an umbrella or parasol has a function similar to a tree because a spherical canopy is supported by a central trunk. However, man-made structures do not generally have as many layers of hierarchy as that found in nature. The main reason for this is the cost and complexity of production. To make a multiple branch parasol would involve a very complex design.

5. Structural features in the main trunk

The trunk is a critical part of a tree. If the trunk suffers from a total breakage, then the tree is likely to die. The trunk is also one of the defining features of trees. Trees are the only type of plant to have a woody trunk. The main structural features of the trunk on a macro level are the tapering and the pre-stressing.

5.1. Tapered trunk

The trunk is tapered in diameter from a maximum at the bottom to a minimum at the top as shown schematically in figure 2. This is efficient because the bending moment also varies from a maximum at the bottom to a minimum at the top of the trunk. To appreciate the effect of tapering, consider an ideal trunk that tapers in diameter in a linear way from d_t at the bottom of the trunk to $d_t/2$ at the top of the trunk. Such a taper has 25/64 of the mass of a cylindrical trunk of constant diameter d_t, which results in a mass saving of about 61%. This example shows that tapering makes a significant mass saving for a tree.

A lamppost is a very common example of an engineering structure that is usually tapered along its length. It is interesting to note that a lamppost is similar in orientation and height to a moderately sized tree trunk. Lampposts are produced in such great quantities that tapering is a very important means of saving mass.

5.2. Residual stresses in the trunk

Most tree trunks have residual tensile stresses in their outer fibres (Gordon 1978) along the axis of the tree. When the tree is subjected to aerodynamic loading, bending stresses are superimposed on the residual tensile stresses. Pre-stressing is a beneficial structural feature because when the trunk is subjected to bending moments, the net compressive stresses are less than the net tensile stresses. Since the compressive strength of wood is lower than the tensile strength, the preloading significantly improves the strength of the tree. This causes a significant reduction in the net compressive stresses experienced by the tree.

Pre-stressing is common in engineering structures such as concrete beams. Man-made pre-stressed structures are usually made from composite materials such as concrete and steel. This contrasts with natural structures like wood where pre-stressing occurs within the same material. However, some engineering components do have residual stresses within one single material. For example, the surfaces of some holes and shafts are sometimes compressed in order to introduce residual stresses.

6. Structural features in the main branches

The main branches of trees have the function of connecting the smaller branches of the canopy to the trunk. The main branches are sometimes subjected to very high loads. One reason for this is that a single main branch may support a large part of the leaf canopy and associated branch network. Another reason is that main branches are often horizontal and therefore are subjected to bending loads caused by self-weight. Like the main trunk, main branches are tapered. Other interesting structural features include efficient beam sections, reinforced joints and struts.

6.1. Rectangular sections

Measurements of the cross-sectional shapes of horizontal branches were carried out. It was found that as branch sections became larger, they became deeper and more rectangular in section. Figure 3 shows an example of a cross-section of a horizontal branch of a pine tree. The branch section is approximately 19.5 cm wide and 29.5 cm deep. In figure 3 it can be seen that the branch originated near the top of the cross-section. As the branch has grown in size, it has grown downwards where compressive stresses are highest.

The reason why horizontal branches change from circular to rectangular sections as the branch grows larger is that bending due to self-weight becomes more important than bending due to wind loading, as explained in section 3. Since a tree responds to compressive stresses by growing more cells in that area, and since the maximum compressive stresses due to self-weight occur on the under side of horizontal branches, the result is that horizontal branches grow downwards and become deep rectangular sections.

Figure 3. Cross-section of a large horizontal branch showing adaptive growth.

The following analysis derives a performance factor that compares the efficiency of a deep rectangular section with a round circular section in order to assess the importance of this structural feature. The performance factor, P, is defined such that:

$$P = \frac{m_0}{m_\mathrm{f}} \tag{11}$$

where m_0 is the mass of a part without a particular structural feature and m_f is the mass of the same part when it contains the structural feature. The performance factor P therefore gives a direct indication of the mass saving.

From equation (3) it can be seen that strength of a beam is proportional to the section modulus, Z. The performance factor is equal to the ratio of the mass of the circular section to the rectangular section that have the same section modulus. This ratio is equal to the ratios of the areas. Therefore:

$$P = \frac{\pi d^2}{4bh} \tag{12}$$

For a given strength, the section modulus of a rectangular section is the same as a circular section, therefore:

$$\frac{bh^2}{6} = \frac{\pi d^3}{32} \tag{13}$$

Therefore, from equations (12) and (13), the performance factor is:

$$p = 0.763\left[\frac{\pi h}{b}\right]^{1/3} \tag{14}$$

The largest horizontal branch cross-section measured was 910 mm deep × 320 mm wide. Inserting these values into equation (13) gives a performance factor of $P = 1.58$. This means that this horizontal branch is around 37% lighter than it would be if it retained a circular cross-section.

Deep beam sections are very common in engineering. However, adaptive growth is not common. Adaptive growth is not so relevant to engineering structures because engineering structures do not gradually grow in the way that a tree does. However, some forms of adaptation could be useful for when structures undergo a change in requirements such as the upgrading of a bridge to meet new load requirements. In such cases, if the designers provide space for strengthening of beams, then this could help to make upgrading feasible.

6.2. Reinforced joints

An important feature of branches is that they have reinforcement at their junctions with the main trunk and also with other branches. The reinforcements take the form of large radii. The reinforcements enable a smooth force flow from sub-branches into branches and from the main branches into the trunk. Without such blending there can be a very large stress concentration due to the abrupt change of section. Stress concentration factors of up to about 3 are experienced when there is no radius on a change of section of a shaft (Norton 1996). Therefore, it is likely that the use of radii has an important impact on the strength and efficiency of branches.

6.3. Struts

Some trees use struts to reinforce the strength of large horizontal branches. As the tree grows large, roots are dropped down from horizontal branches and these penetrate the ground and form supports. The Banyan fig tree of South East Asia is one of the most well-known examples of a tree with struts supporting large branches.

Struts can have a substantial effect on the structural efficiency of a cantilever. The following analysis derives a performance factor for the effect of a single strut placed at the tip of a cantilever that gives a simple support. The maximum moment of a cantilever of length L with a uniformly distributed load of w per unit length, is $wL^2/2$. However, if the cantilever is supported at the tip by a simple support, then the maximum bending moment is only $wL^2/8$. This represents a reduction by a factor of 4. This corresponds to a performance factor of $P = 4$. In both cases, there is a linear decrease in bending moment along the beam from a maximum at the root of the cantilever to zero at the tip of the cantilever.

In practice, it is not possible for the tree to get the full benefit of the strut because the strut is limited in strength and does not provide a perfect support. However, the presented analysis shows that struts have the potential to achieve very significant weight savings for the tree.

7. Structural features in the small branches and leaves

In general, structural design involves either designing for strength or designing for stiffness. In the case of trees, the structural features are designed for strength. Strength design has the advantage that it represents a less severe requirement than stiffness design and therefore requires less material. A second advantage of strength design is that it allows a high degree of flexibility and this can help to minimize aerodynamic loading as explained in the following section.

7.1. Flexibility

The small branches and leaves of trees are very flexible and will deform significantly under the action of wind loading. A structure of a typical leaf is shown in figure 4. The stem has a lengthwise groove that has a low torsional stiffness but high bending stiffness. This deformation is a very effective way of reducing aerodynamic loading. The aerodynamic force on a tree is proportional to the frontal area and so any reduction in the area results in a direct reduction of loading.

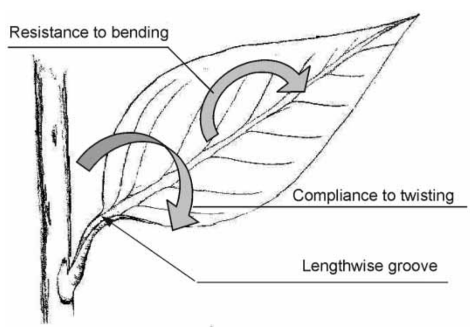

Figure 4. Typical leaf structure.

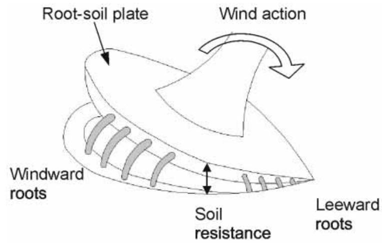

Figure 5. Schematic of root structure.

Small branches deform because they are small section. However, leaves have a particular feature that helps them to distort in the wind. The stem of leaves has a groove that gives relatively high bending stiffness but relatively low torsional stiffness. Bending stiffness is important so that the leaves can support their own weight plus moisture. But low torsional stiffness helps the leaves to give way when the wind is blowing on them.

8. Structural features in the roots

8.1. Root system

Trees grow lateral roots outwards to make a firm anchor to the ground, as shown in figure 5. The outward roots support a soil plate that creates a large weight that helps hold the tree in place. The roots also have a large surface area with the soil and this interface provides a degree of shear strength. As well as lateral roots, the tree also grows deep sinker roots. These roots have a very important function of reaching water in dry weather. However, these roots also add significant stiffness to the root system.

An engineering analogy to roots is shown in figure 6. This diagram shows a column with hold down bolts that are cast in concrete in the ground to make a strong anchorage.

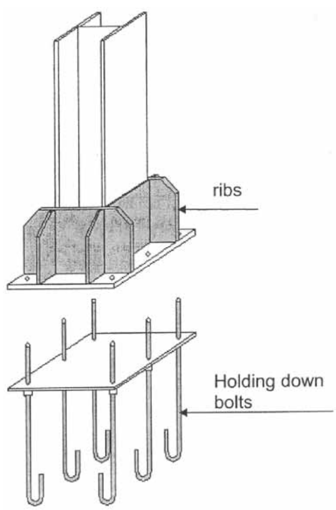

Figure 6. Engineering anchoring system.

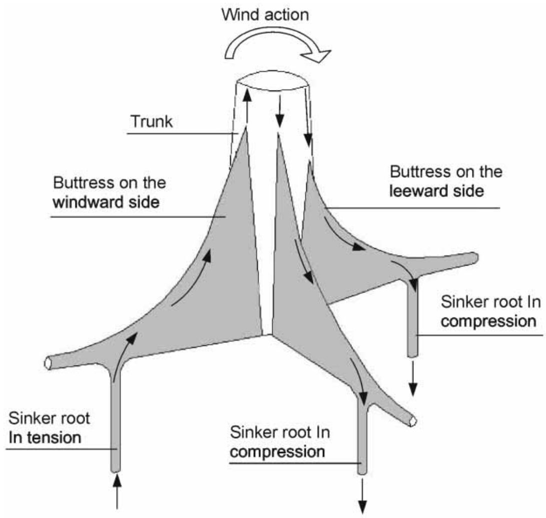

Figure 7. Buttresses at root of trunk.

8.2. Buttresses

With very fast growing trees such as those in tropical rain forests, trees grow a system of buttresses as shown in figure 7 (Mattheck 1991, Ennos 1995). The buttresses brace the trunk like angle brackets. The function of the buttresses is to strengthen the roots. An engineering analogy to the buttresses on trunks is shown in figure 6, which shows angle brackets strengthening the column.

9. Micro-structure

The micro-structure of wood has a very significant impact on the structural efficiency of the trunk and branches. A typical microstructure consists of regular hollow cells typically 50 μm in size and typically hexagonal in shape (Gibson *et al.* 1995). The micro-structure has the effect of reducing the bulk density of a material. In the case of the bending of a beam, the micro-structure has the effect of pushing material away from the neutral axis and this makes the beam more efficient.

The following analysis assesses the effect of adding a micro-structure to a solid circular shaft that has to meet a given bending moment requirement, M. First, consider a solid circular shaft of diameter d_s and density ρ_s. Now consider a second shaft of equivalent bending strength that contains the same basic material but has a micro-structure in the form of cellular holes. This second shaft has a diameter of d_m that is larger than d_s and it has a bulk density ρ_m that is less than ρ_s.

For a given strength requirement the performance factor of efficiency is given by:

$$P = \frac{m_s}{m_m} \tag{15}$$

where m_s is the mass of a solid circular beam and m_m is the mass of the micro-structured circular beam. The bulk density of the micro-structured material is given by:

$$\rho_m = v\rho_s \tag{16}$$

	Balsa	Spruce	Pine	Ash	Lignum
Volumetric fraction, v	0.1	0.25	0.4	0.5	0.7
Performance factor, P	2.2	1.6	1.4	1.3	1.1

Table 3. Performance factor for modelling the effect of micro-structure.

where v is the volumetric fraction of the solid material in the micro-structured material. Since the mass of a beam is directly proportional to the cross-sectional area, the performance factor is given by:

$$P = \frac{d_s^2}{v d_m^2} \tag{17}$$

The strength of a beam for a bending moment requirement is given by equation (3). Since we are comparing two beams that meet the same bending moment requirement, M, the following relationship exists:

$$\sigma_m Z_m = \sigma_s Z_s \tag{18}$$

The strength of the micro-structured material is related to the strength of the solid material by the volumetric faction as follows:

$$\sigma_m = \sigma_s v_s \tag{19}$$

Therefore, combining equations (18) and (19) and noting that the section modulus $Z = \pi d^3/32$, this gives the following expression between the diameters of the two beams:

$$d_s = d_m v^{1/3} \tag{20}$$

Combining equations (17) and (20) gives the following equation for the performance factor:

$$P = \frac{1}{v^{1/3}} \tag{21}$$

Table 3 presents the volumetric fraction of a range of woods together with the performance factor that models the effect of micro-structure on bending strength.

The results in table 3 show that micro-structuring has a significant effect on increasing the strength of the material. In addition, the table shows that materials with a high volume of space (low volumetric fraction of material) have the highest gains in strength. In particular, balsa wood experiences a dramatic increase in strength due to its very low bulk density. However, it should be noted that woods like balsa have a drawback of having a relatively low hardness and therefore low level of robustness.

10. Discussion

Trees contain many different structural features so that a minimum of material is used in the whole structure. The structural efficiency of the tree means that it can grow in height very fast and with minimal feeding requirements. These aspects of performance enable trees to compete effectively for sunlight with other trees.

As has been shown in each section, engineers employ virtually all of the structural features found in trees. This is not surprising because structural engineering has been practiced and refined for thousands of years. In addition, engineers have been fully aware of the macro-level features of trees.

Despite the similarities between the structural features of trees and the structural features of man-made structures, there are still some aspects of design that are mainly unique to trees:

i. *Multi-functionality.* Some of the key structural features of trees have important non-structural functions. For example, the micro-structure of wood has the function of drawing up water by capillary action as well as providing structural efficiency. The hierarchical root system has the function of drawing up water by osmosis as well as providing a secure foundation. This aspect of design contrasts with engineering structures where designers have traditionally separated functions as much as possible. Nature shows that a really optimal design may sometimes involve a very close integration of functions.
ii. *Adaptable growth.* Adaptive growth is a very important process to a tree. A tree can respond to changes in loading so that it grows in the most efficient way. Trees show that smart structures can be a very useful means of producing an optimum design.
iii. *Highly refined hierarchy.* A large tree has many layers of hierarchy in the roots, branches, leaves and even within the wood itself. Trees show that structural hierarchy plays an important role in producing an optimum structural design. In addition, trees indicate that a highly optimized structure requires a hierarchy that is highly refined (i.e. contains many layers).

11. Conclusions

Trees are remarkable structures that provide an elegant and efficient solution to the problem of connecting a central column with a canopy that has a large surface area. Structural efficiency is important to a tree and can make the difference between survival and extinction. Trees contain many of the structural features employed by engineers. However, there are a few aspects of design that are mostly unique to trees. These include the existence of important non-structural functions in structural members, the presence of adaptive growth and the presence of highly refined structural hierarchies. Trees provide an important insight into how future highly optimized structures might be designed. In particular, multi-functioning structures with smart adaptable behaviour may be a feature of structural designs in the future.

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